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On the practical application and control energy efficiency of ISMC: A comparison with traditional SMC in second-order systems

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Abstract
Integral Sliding Mode Control (ISMC) has emerged as an advanced alternative to classical Sliding Mode Control (SMC), offering improvements in chattering reduction and robustness to uncertainties and disturbances. While numerous studies have explored the application of ISMC in various systems, a comprehensive understanding of its control energy consumption, especially in systems with bounded control inputs, remains limited. This paper aims to address this gap by conducting a comparative analysis of the control energy expenditure between ISMC and traditional SMC. The investigation will focus on a fundamental and widely used system: a second-order drive. By directly comparing the energy requirements of both control strategies in this context, this research seeks to provide valuable insights into the practical applicability and efficiency of ISMC, particularly concerning its potential for increased energy demand due to the integral sliding surface design. The findings of this study will contribute to a more informed selection of appropriate control methodologies for various engineering applications.

Keywords: SMC, ISMC, sliding mode, integral sliding mode, robustness, convergence time.

1. Introduction

Integral Sliding Mode Control (ISMC) represents an advanced variant of classical Sliding Mode Control (SMC), specifically designed to mitigate common limitations such as chattering and the restricted adaptability to uncertainties and external disturbances. Distinct from traditional SMC, ISMC defines the sliding surface from the initial time instant, thereby ensuring the system's invariance to uncertainties and disturbances from the onset. The integral component within the sliding surface enhances steady-state accuracy, reduces tracking errors, and improves overall control performance, particularly in nonlinear and uncertain systems. Despite these advantages, recent research applying the ISMC controller has revealed certain persistent limitations.

For instance, Zhang *et al.* [1] proposed an adaptive ISMC approach for highly uncertain nonlinear systems, demonstrating enhanced robustness and responsiveness. However, the adaptive parameter tuning increases computational complexity, potentially hindering real-time implementation. Similarly, Zhao *et al.* [2] introduced a finite-time ISMC scheme for underactuated systems to achieve rapid convergence. Nevertheless, this method necessitates high-order derivative estimation, which can be susceptible to measurement noise in practical applications. Sharma and Kumar [3] applied ISMC to enhance energy harvesting in PV systems under partial shading conditions. Their work, however, did not account for dynamic variations in the DC-DC converter, which could impact real-time control accuracy. Furthermore, Zhou *et al.* [4] addressed packet dropout issues in networked control systems using ISMC to achieve robust performance. Their analysis assumes Bernoulli-distributed packet loss, which may not accurately represent the complexities of real-world network behavior. Zhang *et al.* [5] explored the integration of terminal sliding mode with ISMC to achieve faster convergence in robotic manipulators. However, they did not consider the effects of nonlinear friction in the actuators, which can negatively affect tracking accuracy. Lastly, Meng *et al.* [6] combined T–S fuzzy modeling with ISMC to manage significant system nonlinearities. Yet, the system performance heavily depends on expert knowledge for the selection of fuzzy rules and membership functions, which can limit its general applicability.

To further validate the practical application of this ISMC control algorithm, more detailed investigations into its effectiveness are necessary. This is particularly crucial in systems with bounded control inputs. The reason for this lies in the fact that the design and maintenance of the integral sliding surface might inherently demand more control energy compared to traditional sliding surfaces. Consequently, a comprehensive study is required, focusing on fundamental and prevalent systems such as second-order drive systems, to analyze the control energy expenditure and provide a direct comparison with conventional sliding mode control. This research aims to address this gap.

Methodology

A. Conventional SMC for a DC motor

1. Mathematic model of a DC motor.

Then, the mathematical model of the DC motor is given by Equation 1.

$$\begin{Bmatrix} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x,t) + B(x,t) * u + d(t) = -ax_1 - bx_2 + u + d(t) \end{Bmatrix} \quad (1)$$

Where

- x_1, x_2 are angle and angular velocity of the motor shaft
- *a*, *b* are positive parameters.
- *u:* is control input of the DC motor
- *d(t):* disturbances that affect the system.

2. Traditional SMC.

Assume the system needs to track the desired values $x_d, \dot{x}_d$, i.e., ensure the tracking errors $e_1 = x_1 - x_d \to 0$ and $e_2 = x_2 - \dot{x}_d \to 0$. Then, the conventional sliding variable has the form:

$$S = \lambda e_1 + e_2 \quad (2)$$

Then the derivative of the sliding variable has the form:

$$\dot{S} = \lambda \dot{e}_1 + \dot{e}_2 = \lambda(\dot{x}_1 - \dot{x}_d) + (\dot{x}_2 - \ddot{x}_d) \quad (3)$$

Substituting (1) into (3) we have:

$$\dot{S} = \lambda x_2 - \lambda \dot{x}_d - \ddot{x}_d - ax_1 - bx_2 + u + d(t) \quad (4)$$

Then with the conventional sliding mode controller, the control input consists of 2 components, the equivalent control and the discontinuous control:

$$u = u_{eq} + u_d \quad (5)$$

Where, the equivalent component helps maintain the sliding variable S on the sliding surface S=0, or ensures:

$$\dot{S} = 0 \quad (6)$$

Substituting into (4) we find u_{eq}:

$$u_{eq} = ax_1 + bx_2 + \lambda \dot{x}_d + \ddot{x}_d - \lambda x_2 \quad (7)$$

Meanwhile, the discontinuous control input helps pull the sliding variable towards the sliding surface:

$$u_d = -K\text{sign}(S) \quad (8)$$

B. Integral Sliding mode control (ISMC):

1. Integral Sliding mode control (ISMC):

The ISMC controller is designed with 2 components, the nominal control u_0 ensuring that the system can operate under ideal conditions (no disturbance) and the sliding control component u_{smc}:

$$u = u_0 + u_{smc} \quad (9)$$

In the system given by equation (1), we define u_0 including a feedforward component and a feedback component. Where the feedback uses a PD controller. Then u_0 is determined by the following relation:

$$u_0 = u_{ff} + u_{fb} = (\ddot{x}_d + ax_d + b\dot{x}_d) - (k_p e_1 + k_d e_2) \quad (10)$$

Meanwhile, the sliding control component is built based on a sliding variable different from the conventional sliding variable, in which there is an integral component z:

$$S = S_0 + z \quad (11)$$

Where, S_0 is the conventional sliding variable similar to (2):

$$S_0 = \lambda e_1 + e_2 = \lambda(x_1 - x_d) + (x_2 - \dot{x}_d) \quad (12)$$

And the auxiliary variable z is calculated from the integral of its derivative $\dot{z}$ in nominal system:

$$\dot{z} = -\left[\frac{\partial S_0}{\partial x}\right]\left[\begin{matrix}\dot{x}\\ f(x,t) + B(x,t) * u_0\end{matrix}\right] + \lambda\dot{x}_d + \ddot{x}_d \quad (13)$$

Substituting (1), (10) into (13), we have:

$$\dot{z} = -[\lambda\ 1]\left[\begin{matrix}x_2\\ -ax_1 - bx_2 + u_0\end{matrix}\right] + \lambda\dot{x}_d + \ddot{x}_d = -\lambda x_2 + ax_1 + bx_2 - u_0 + \lambda\dot{x}_d + \ddot{x}_d \quad (14)$$

To calculate the value of z from its derivative, an initial value is needed, and the initial value of z is chosen to be exactly the opposite of the initial value of S_0 to ensure the condition $S_{(t=0)} = 0$, or:

$$z_{(t=0)} = -S_{0_{(t=0)}} = -\lambda(x_{10} - x_{d0}) - (x_{20} - \dot{x}_{d0}) \quad (15)$$

Then (11) becomes:

$$S = S_0 + \int_o^t \dot{z}d\tau \quad (16)$$

And the sliding control component is determined by:

$$u_{smc} = -K\text{sign}(S) \quad (17)$$

2. Analyze stability

A commonly chosen Lyapunov function is a positive definite function related to the sliding variable S. A popular choice is:

$$V = \frac{1}{2}S^2 \quad (18)$$

Since $S^2 \geq 0$ and $V = 0$ only when $s = 0$, this Lyapunov function is positive semi-definite. To prove stability, we need to show that the time derivative of V (denoted as $\dot{V}$) is negative semi-definite, i.e., $\dot{V} \leq 0$.

Taking the time derivative of V, we get:

$$\dot{V} = S\dot{S} \quad (19)$$

From (16) the time derivative of s is:

$$\dot{S} = \dot{S}_0 + \dot{z} \quad (20)$$

Substitute equation (12) and (14) to (20), we have:

$$\dot{S} = \lambda(x_2 - \dot{x}_d) + (\dot{x}_2 - \ddot{x}_d) - \lambda x_2 + ax_1 + bx_2 - u_0 + \lambda\dot{x}_d + \ddot{x}_d \quad (21)$$

Applying (1), (9), (17) to (21):

$$\dot{S} = -ax_1 - bx_2 + u + d(t) + ax_1 + bx_2 - u_0 = -K\text{sign}(S) + d(t) \quad (22)$$

In system (1) the disturbances $d(t)$ is bounded:

$$|d(t)| \leq D \quad (23)$$

Applying to (22) we have:

$$\dot{V} = S\dot{S} = -K\text{sign}(S)S + d(t)S \leq -K|S| + D|S| \quad (24)$$

If $K > D$ then $\dot{V} < 0$ or the system is stable by Lyapunov.

C. Simulations

The simulation process is taken in Matlab Simulink environment with the parameters are given in (25):

$$\begin{cases} a = 1 \\ b = 2 \\ \lambda = 5 \\ D = 0.5 \\ k_p = 10 \\ k_d = 5 \\ K = 1.5 \end{cases} \quad (25)$$

The desired value is assumed to be a sinusoidal signal given by the following equations:

$$\begin{cases} x_d = \sin(t) \\ \dot{x}_d = \cos(t) \\ \ddot{x}_d = -\sin(t) \end{cases} \quad (26)$$

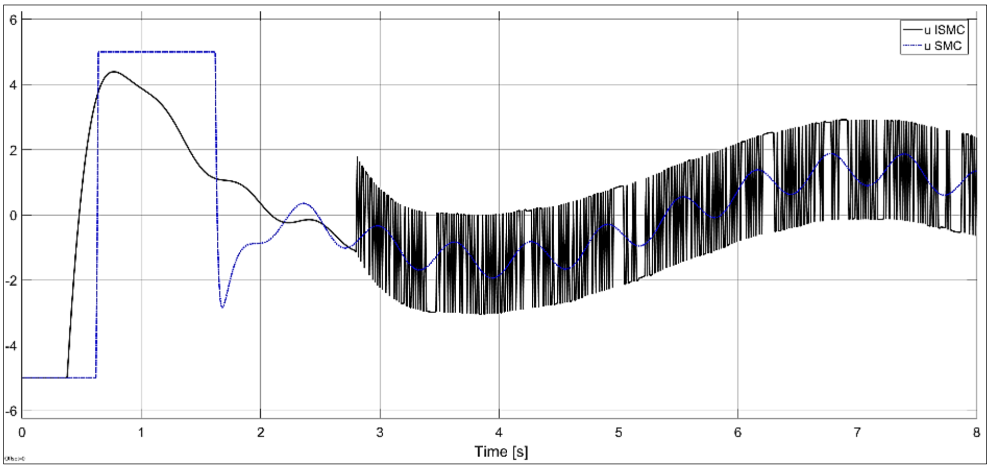
The disturbance acting on the system is assumed to be sinusoidal with amplitude D given above and a frequency ten times higher than the desired signal:

$$d(t) = 0.5\sin(10t) \quad (27)$$

To compare the two control method SMC and ISMC, choose to bound control input by different values: 5, 10, 20, 30.

Results and Discussion

The simulation results are shown in Fig.1 for the case control input is bounded by 5.



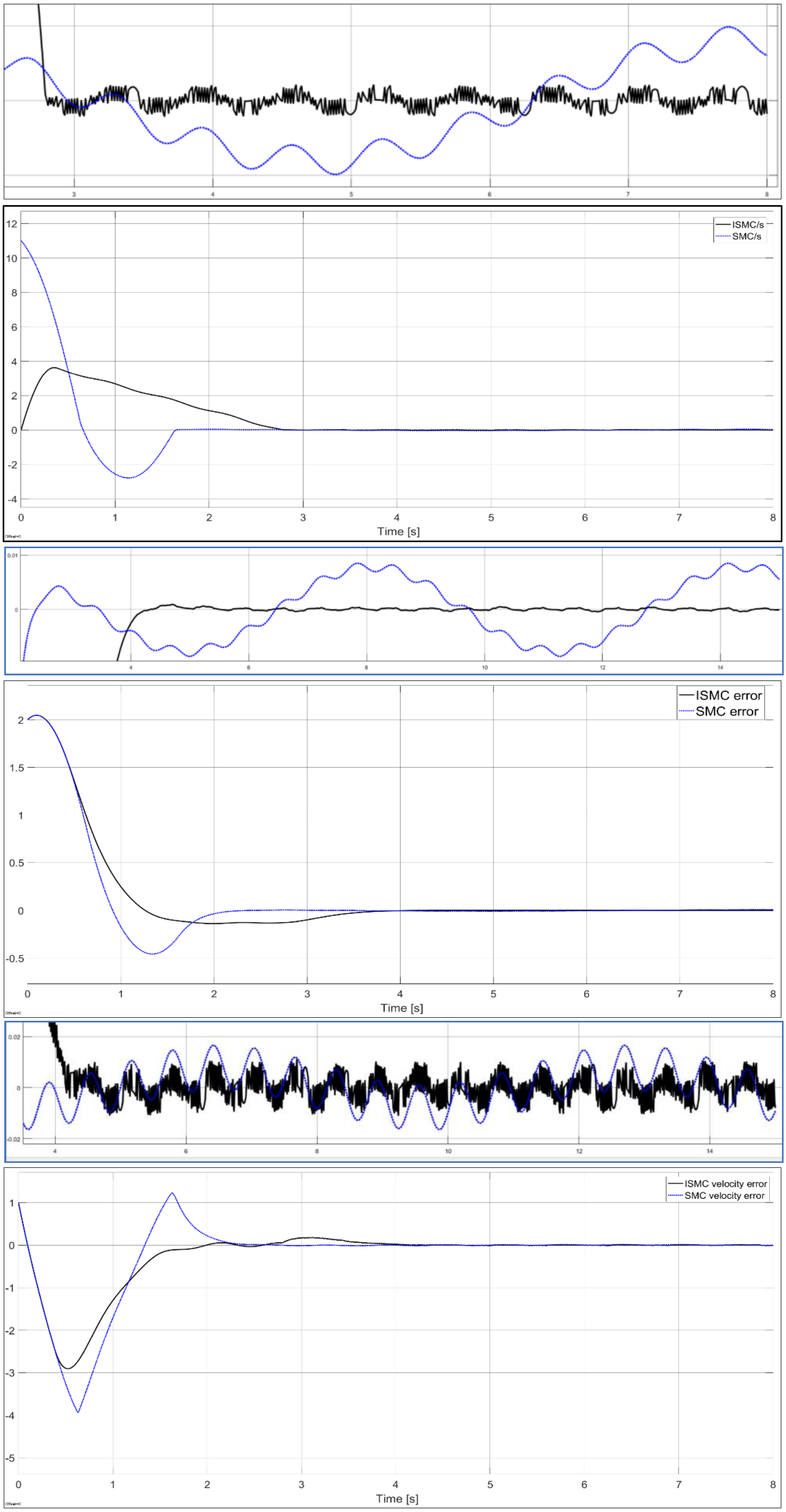
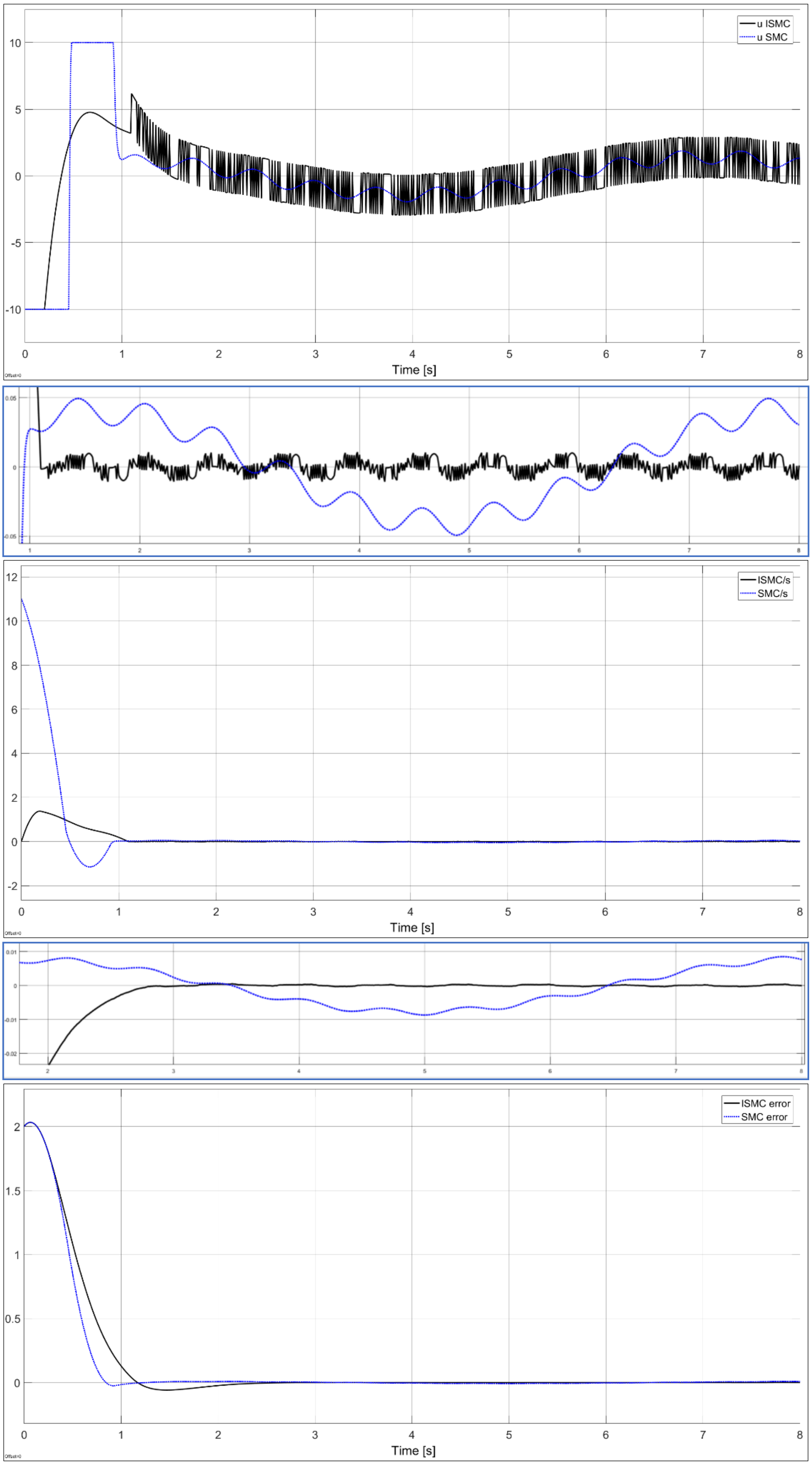


Fig 1: Simulation results when control input is bounded by 5.

The simulation results are shown in Fig.2 for the case control input is bounded by 10.



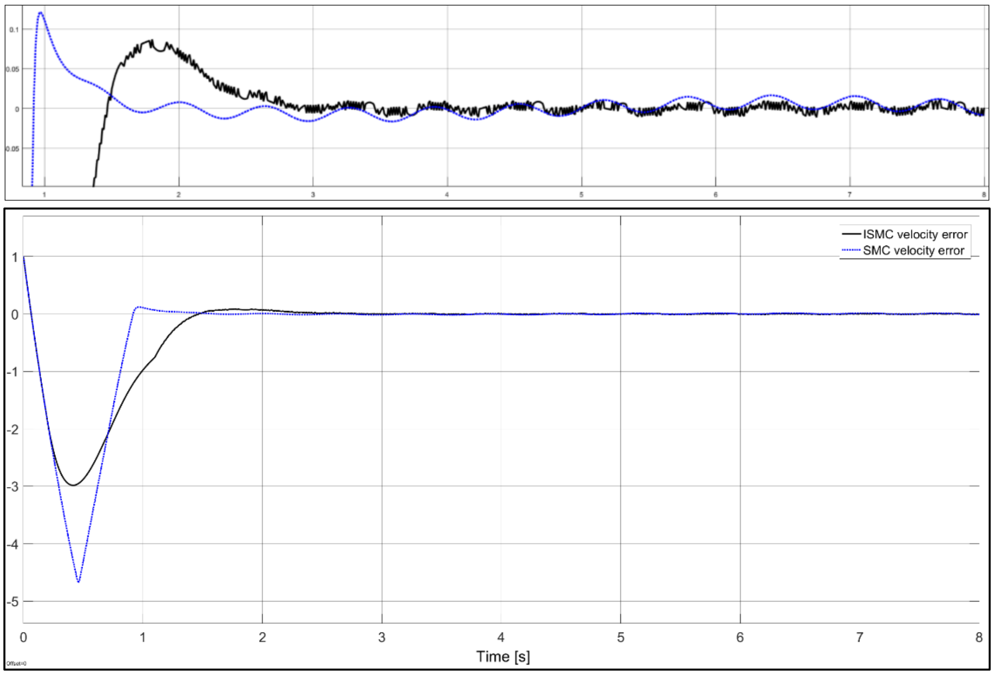
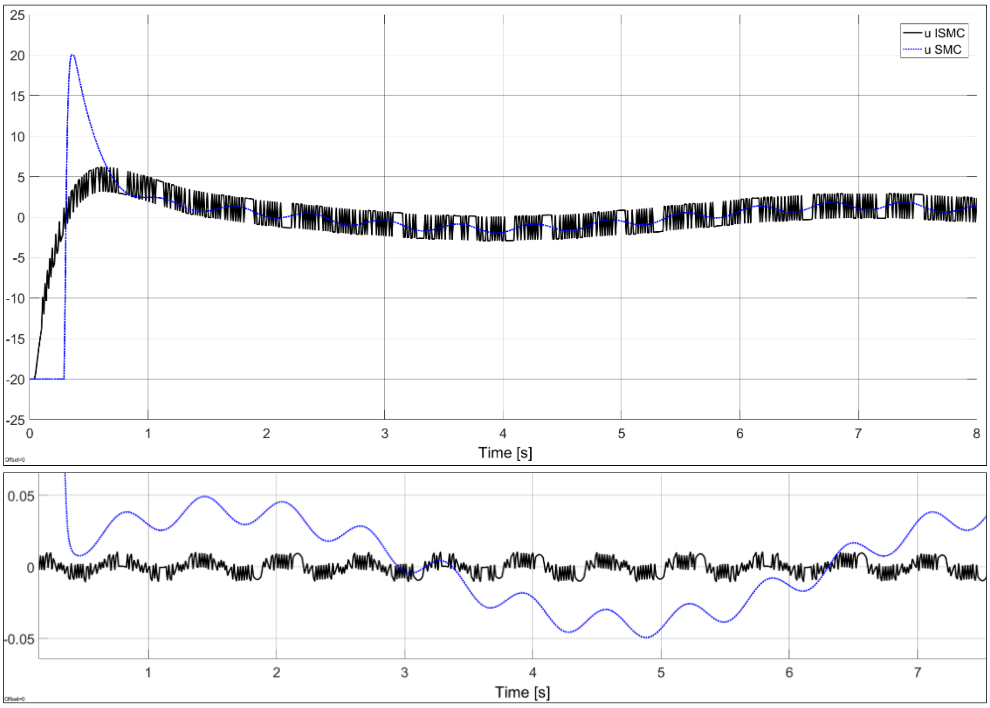


Fig 2: Simulation results when control input is bounded by 10.

The simulation results are shown in Fig.3 for the case control input is bounded by 20.



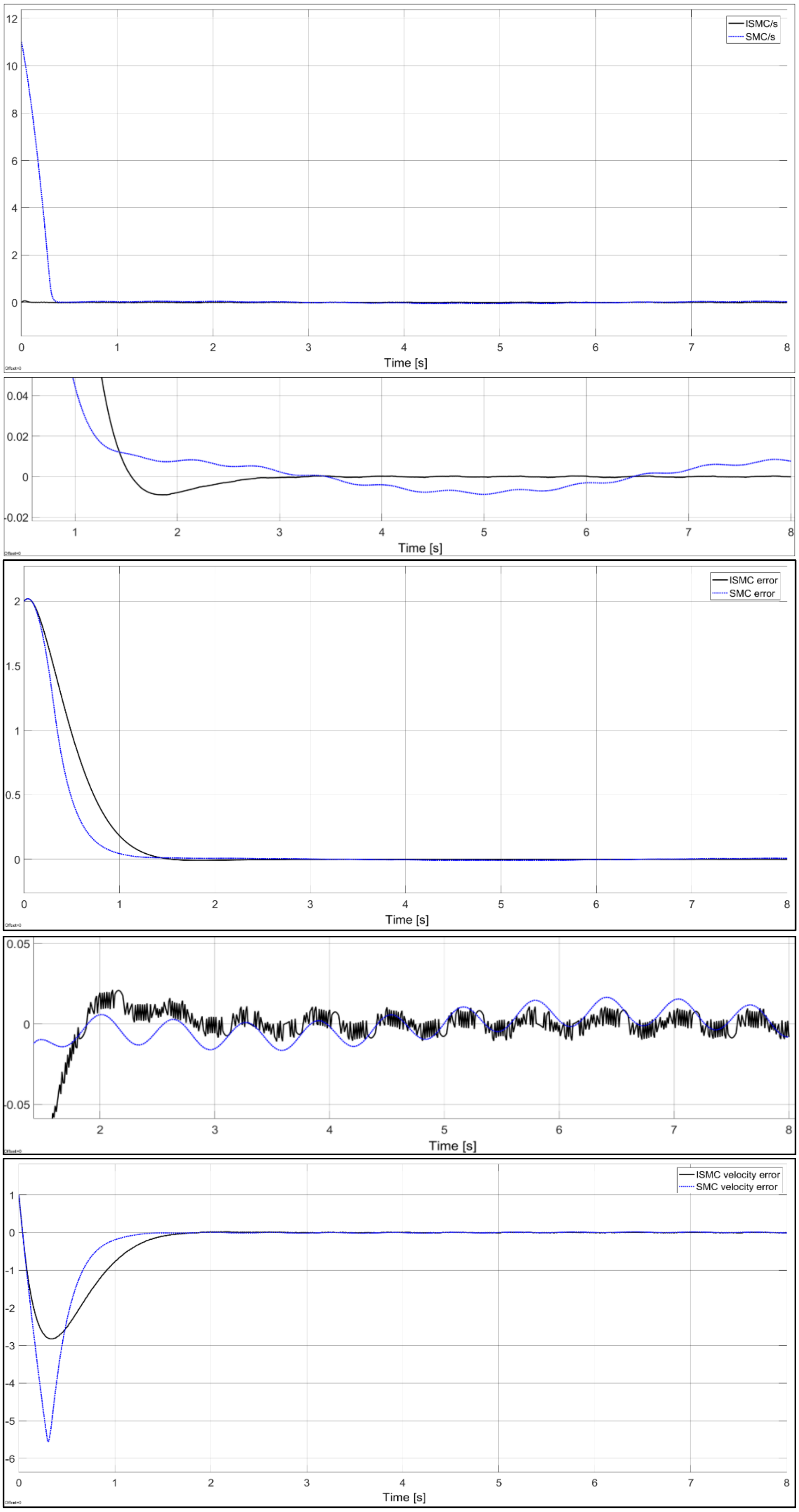
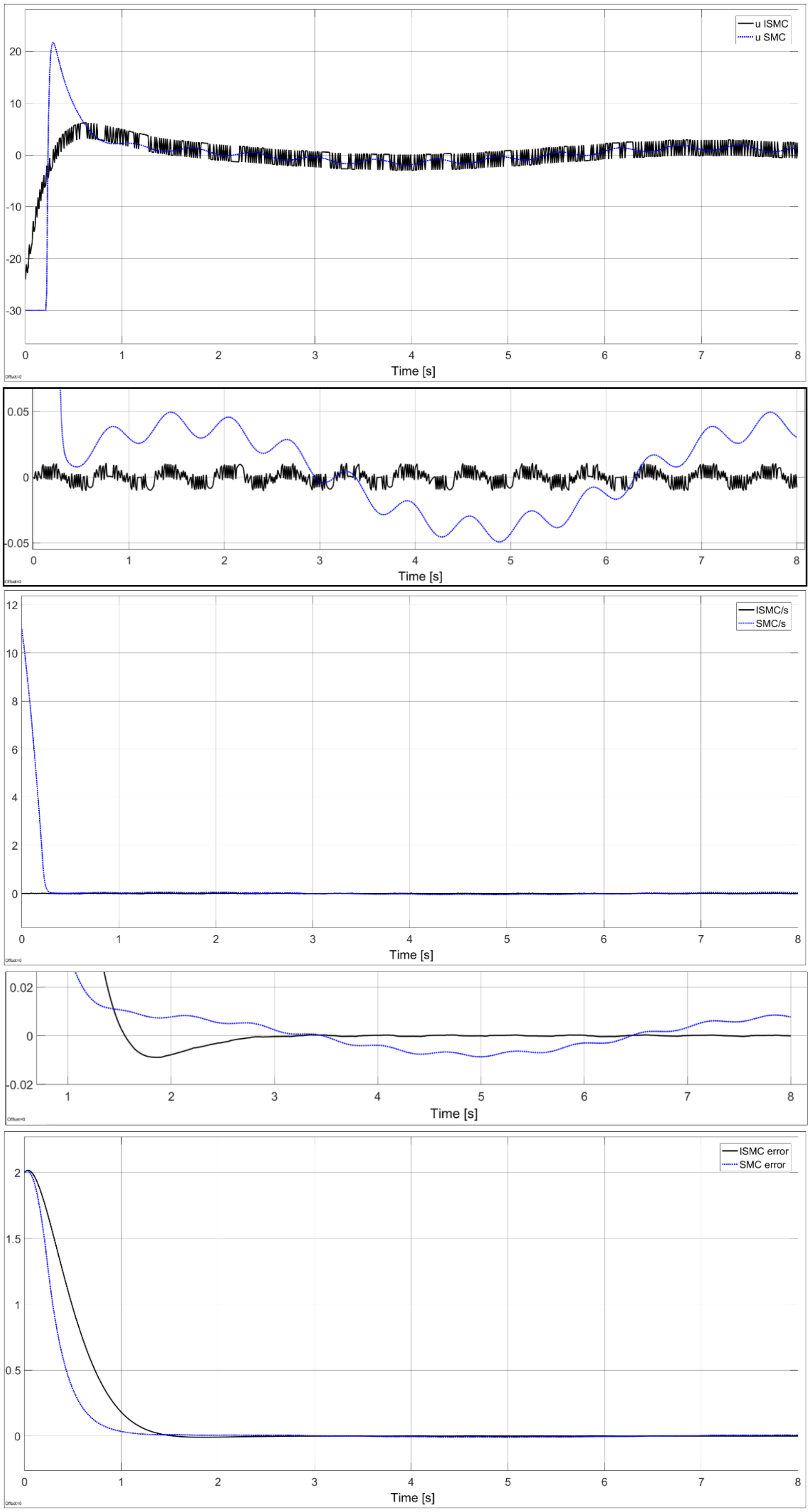


Fig 3: Simulation results when control input is bounded by 20.

The simulation results are shown in Fig.4 for the case control input is bounded by 30.



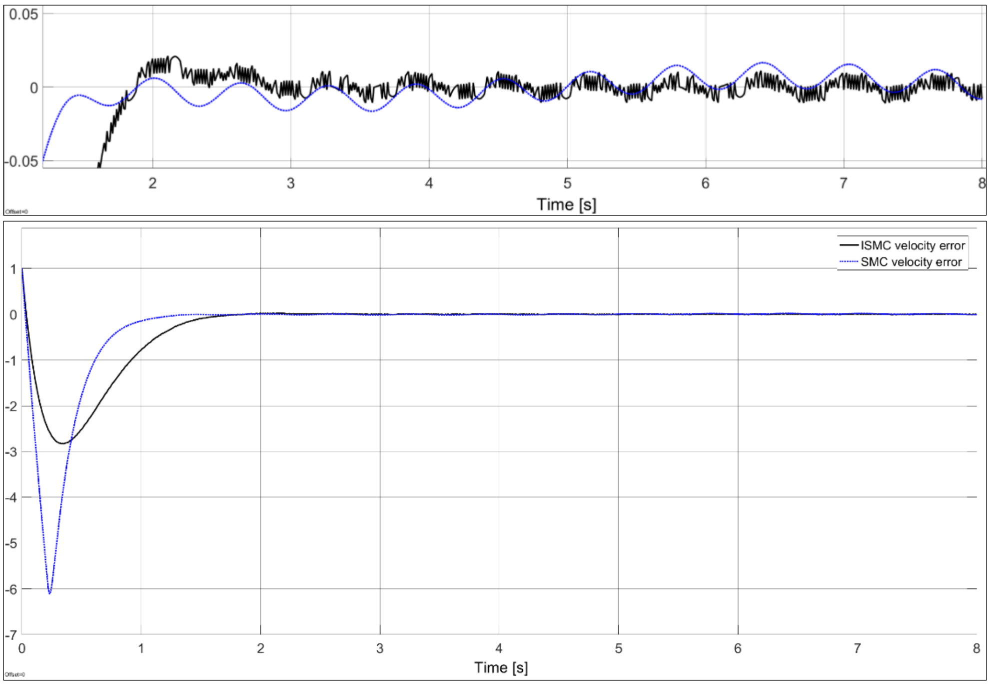


Fig 4: Simulation results when control input is bounded by 30.

The results in Fig. 1 and Fig. 2 show the cases when control inputs are not enough to keep the ISMC sliding variable *S* on the sliding surface. Then it has to fall out of the sliding surface a little bit before reaching back on to the sliding surface. When the cases in Fig. 3 and Fig. 4 the control input are enough for the job, especially in Fig. 4. Then the quality of the system in those cases are also different. Convergence time of the system in Fig. 1 and Fig. 2 with SMC are shorter compared to ISMC when in Fig. 3 and Fig. 4 the convergence time are close to each other. So the ISMC does not improve the convergence time compared to conventional SMC.

However the impact of ISMC are clearly shown in the robustness against disturbances. In the four Figures the sliding variable *S* of SMC are fluctuated around 0 or it is not actually on the sliding surface but staying around the surface. Then the state errors are also fluctuated around 0 as they are shown in the third and forth figure of each Figures. Both states including position error and velocity error have the problem, but the fluctuation of position error are larger.

Conclusions

This article has compared the ISMC method to conventional SMC. The simulation, comparison and evaluation results have shown the advantages and disadvantages of the two methods, which are:

- The ISMC method has the advantage of reducing the impact of disturbances on the system compared to conventional SMC.
- The ISMC method requires greater control input compared to conventional SMC method. It also has larger convergence time compared to conventional SMC, especially when the control input is small.

Therefore, users need to choose the required system performance when using ISMC or SMC. When a large amount of energy can be used and the system needs to be stable against disturbances, ISMC is a good option. Conversely, when resources are limited and fast convergence time is required, ISMC is not a better option than conventional SMC.

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