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Determining UAV Coordinates from Ground Transmitting Stations

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Abstract

Navigation for UAVs to follow a trajectory requires determining their coordinates. Common methods include using GPS or INS or both. However, these systems have certain drawbacks, and a supplementary method is always necessary to ensure system reliability in all cases. This paper develops a positioning method based on time synchronization from broadcasting points with given coordinates to determine UAV coordinates, builds a specific algorithm to determine UAV coordinates from three ground signal transmitting stations, and provides an illustrative example, thereby demonstrating the effectiveness of the proposed method.

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Introduction

In UAV control, positioning the UAV to ensure its navigation according to certain requirements is an important problem. Typically, navigation and positioning systems are built upon GPS and INS. These systems have the advantage of being compact and easy to implement. However, GPS signals can sometimes be unusable due to interference or poor signal quality, and INS systems always accumulate errors over time. One solution to overcome these drawbacks is a positioning method based on time synchronization from standard broadcasting points. This method relies on the time difference of arrival of electromagnetic waves transmitted from broadcasting points with known coordinates to the receiving point.

In practice, the time difference of arrival (TDOA) positioning method has been applied to radar systems as well as mobile communication systems, where multiple receiving stations are used to detect signal sources. It is applied to determine the coordinates of moving objects using ground-based means, and the active signal source originates from the moving objects. The positioning method based on the time difference of arrival of electromagnetic waves transmitted from broadcasting points with known coordinates is like an inversion of the TDOA positioning method from a single broadcasting point to receiving points on the ground. This method allows the UAV's control system to determine its own coordinates.

To solve the problem of processing TDOA signals for ground-based radar receiving points, the signal from the target is complex, unknown, and contains noise, so algorithms are applied to overcome the effects of noise and calculate target coordinates. Unlike passive radar, the UAV coordinate determination system based on the time difference of arrival from ground transmitting points uses known active signals, making processing more straightforward, allowing for precise determination of signal arrival times, and thus enabling clear and explicit calculations. The problem of determining the coordinates of moving objects, specifically underwater vehicles, is also discussed in based on the distance from the underwater vehicle to pre-set landmarks; however, the method for determining the distance from the underwater vehicle to the landmarks is not presented. The proposed algorithm below uses the time difference of arrival from standard landmarks to determine the distance deviation of the UAV to the standard points and thereby determine the UAV's coordinates.

Methodology

A. Method for determining UAV coordinates using 3 ground transmitting points

The essence of this method is that based on the time difference of transmission from the reference landmark and other standard landmarks to the UAV, a system of equations with two unknowns, the UAV's coordinates, can be constructed. Then, solving this system of equations will determine the UAV's coordinates.

1. Problem statement

Assume the coordinates of the reference landmark are $C_0(x_0 \quad y_0 \quad h_0)$ standard landmark 1 is $C_1(x_1, y_1, h_1)$, and standard landmark 2 is $C_2(x_2, y_2, h_2)$. The UAV's coordinates are $P_a(x_a, y_a, h_a)$. Then we have the following system of equations:

$$\begin{cases}\sqrt{(h_a-h_0)^2+(x_a-x_0)^2+(y_a-y_0)^2}-\sqrt{(h_a-h_1)^2+(x_a-x_1)^2+(y_a-y_1)^2}=c*\Delta t_1\\ \sqrt{(h_a-h_0)^2+(x_a-x_0)^2+(y_a-y_0)^2}-\sqrt{(h_a-h_2)^2+(x_a-x_2)^2+(y_a-y_2)^2}=c*\Delta t_2\end{cases} \quad (1)$$

Where c is the speed of light in air, c = 300,000,000 m/s. The
system of equations (1) is a system of two quadratic equations with three unknowns. The system of equations (1) can be solved by an iterative approximation method. To solve it explicitly, choose an arrangement such that the standard points C_0, C_1, C_2 lie on a straight line and have the same altitude. Without loss of generality, choose C_0 as the origin, and the line connecting C_0, C_1 , C_2 as the x-axis. Then the coordinates of the standard points are $C_0(0,0,h_0), C_1(x_1, 0, h_0), C_2(x_2, 0, h_0)$. Assume the UAV's operating area is in the upper half-plane of the coordinate system.

Then we have the following system of equations:

$$\begin{cases}\sqrt{(h_a-h_0)^2+x_a^2+y_a^2}-\sqrt{(h_a-h_0)^2+(x_a-x_1)^2+y_a^2}=c*\Delta t_1=\Delta l_1\\ \sqrt{(h_a-h_0)^2+x_a^2+y_a^2}-\sqrt{(h_a-h_0)^2+(x_a-x_2)^2+y_a^2}=c*\Delta t_2=\Delta l_2\end{cases} \quad (2)$$

For convenience, let $y_a^2+(h_a-h_0)^2=d_a^2$. Then (2) becomes:

$$\begin{cases}\sqrt{d_a^2+x_a^2}-\sqrt{d_a^2+(x_a-x_1)^2}=\Delta l_1\ (3a)\\ \sqrt{d_a^2+x_a^2}-\sqrt{d_a^2+(x_a-x_2)^2}=\Delta l_2\ (3b)\end{cases} \quad (3)$$

Equations (3a) and (3b) are similar. Perform the transformation for equation (3a). First, we rearrange the terms:

$$\sqrt{d_a^2+x_a^2}-\Delta l_1=\sqrt{d_a^2+(x_a-x_1)^2} \quad (4)$$

Squaring both sides:

$$d_a^2+x_a^2+\Delta l_1^2-2\Delta l_1\sqrt{d_a^2+x_a^2}=d_a^2+(x_a-x_1)^2 \quad (5)$$

Further rearrangement yields:

$$2x_a x_1-x_1^2+\Delta l_1^2=2\Delta l_1\sqrt{d_a^2+x_a^2} \quad (6)$$

Squaring both sides:

$$(\Delta l_1^2-x_1^2)^2+4x_1^2x_a^2+4x_1(\Delta l_1^2-x_1^2)x_a=4\Delta l_1^2(d_a^2+x_a^2) \quad (7)$$

Rearranging into a quadratic equation in x_a:

$$(4x_1^2-4\Delta l_1^2)x_a^2+4x_1(\Delta l_1^2-x_1^2)x_a+(\Delta l_1^2-x_1^2)^2-4\Delta l_1^2d_a^2=0 \quad (8)$$

Performing similar transformations for equation (3b) yields:

$$(4x_2^2-4\Delta l_2^2)x_a^2+4x_2(\Delta l_2^2-x_2^2)x_a+(\Delta l_2^2-x_2^2)^2-4\Delta l_2^2d_a^2=0 \quad (9)$$

Multiplying equation (8) by Δl_2^2 and subtracting the product of equation (9) with Δl_1^2 yields a quadratic equation in x_a:$d = v.cos\Delta\psi.\Delta\dot{\psi} = u.\ v.\ cos\Delta\psi$

Set :

$$a=[(4x_1^2-4\Delta l_1^2)\Delta l_2^2-(4x_2^2-4\Delta l_2^2)\Delta l_1^2]$$
$$b=[4x_1(\Delta l_1^2-x_1^2)\Delta l_2^2-4x_2(\Delta l_2^2-x_2^2)\Delta l_1^2]$$
$$c=[(\Delta l_1^2-x_1^2)\Delta l_2^2-(\Delta l_2^2-x_2^2)\Delta l_1^2]$$

We obtain the quadratic equation**:**

$$ax_a^2 + bx_a + c = 0 \quad (10)$$

With solutions:

$$\begin{cases} x_{a1} = \frac{-b+\sqrt{b^2-4ac}}{2a} \\ x_{a2} = \frac{-b-\sqrt{b^2-4ac}}{2a} \end{cases} \quad (11)$$

The question arises whether equation (10) has a positive real solution, and if so, whether it is unique. This is proven below.

2. Mathematical Basis

Lemma 1: The locus of points in the upper half-plane where the difference in distances to two points on the x-axis is constant is a monotonically increasing or decreasing curve.
Proof:

The equation for points whose distance to two points x_0, x_1 is a constant *d* is given by:

$$f(x,y) = \sqrt{y^2 + (x - x_0)^2} - \sqrt{y^2 + (x - x_1)^2} = d = const \quad (12)$$

$$\frac{df(x,y)}{dx} = \left[\frac{1}{2\sqrt{y^2+(x-x_0)^2}}\left(2y\frac{dy}{dx} + 2(x - x_0)\right) - \frac{1}{2\sqrt{y^2+(x-x_1)^2}}\left(2y\frac{dy}{dx} + 2(x - x_1)\right)\right] \quad (13)$$

$$\frac{df(x,y)}{dy} = \left[\frac{1}{2\sqrt{y^2+(x-x_0)^2}}\left(2(x - x_0)\frac{dx}{dy} + 2y\right) - \frac{1}{2\sqrt{y^2+(x-x_1)^2}}\left(2(x - x_1)\frac{dx}{dy} + 2y\right)\right] = 0 \quad (14)$$

Since $f(x,y) = const$

$$\frac{dx}{dy}\left[\frac{(x-x_0)\sqrt{y^2+(x-x_1)^2}-(x-x_1)\sqrt{y^2+(x-x_0)^2}}{\sqrt{y^2+(x-x_0)^2}\sqrt{y^2+(x-x_1)^2}}\right] = \frac{y\sqrt{y^2+(x-x_1)^2}-\sqrt{y^2+(x-x_0)^2}}{\sqrt{y^2+(x-x_0)^2}\sqrt{y^2+(x-x_1)^2}} \quad (15)$$

$$\frac{dx}{dy} = \frac{y\left(\sqrt{y^2+(x-x_1)^2}-\sqrt{y^2+(x-x_0)^2}\right)}{(x-x_0)\sqrt{y^2+(x-x_1)^2}-(x-x_1)\sqrt{y^2+(x-x_0)^2}} \quad (16)$$

Transforming the denominator:

$$(x - x_0)\sqrt{y^2 + (x - x_1)^2} - (x - x_1)\sqrt{y^2 + (x - x_0)^2} =$$
$$= (x - x_0)\left(\sqrt{y^2 + (x - x_1)^2} - \sqrt{y^2 + (x - x_0)^2}\right) + \left((x - x_0) - (x - x_1)\right)\sqrt{y^2 + (x - x_0)^2}$$
$$= -(x - x_1)d + (x_2 - x_1)\sqrt{y^2 + (x - x_1)^2} \quad (17)$$

Combining (16) with (17) and condition (12) yields:

$$\frac{dx}{dy} = \frac{-y.d}{(x_1-x_0)\sqrt{y^2+(x-x_1)^2}-d(x-x_0)} \quad (18)$$

Since $\sqrt{y^2 + (x - x_1)^2} > |x - x_0|$ and $(x_1 - x_0) > |d|$ the denominator is always positive.

Thus, in the upper half-plane, since y>0:

$$\operatorname{sgn}\left(\frac{dx}{dy}\right) = -\operatorname{sgn}(d) \Rightarrow \text{x and y are monotonically increasing or decreasing} \quad (19)$$

The lemma is proven.
Lemma 2: Two monotonically increasing or decreasing curves $y = f_1(x)$ and $= f_2(x)$, if they have derivatives at every point x_i satisfying $\left.\frac{dy}{dx}\right|_{x_i} > \left.\frac{dg}{dx}\right|_{x_i}$ (20) or $\left.\frac{dy}{dx}\right|_{x_i} < \left.\frac{dg}{dx}\right|_{x_i}$ (21), can only intersect at one point.
Proof :
Assume they intersect at two points (x_1, y_1) and (x_2, y_2)
Then: $y_1 = g_1, y_2 = g_2$

and

$$y_2 - y_1 = g_2 - g_1 \tag{22}$$

However, if condition (20) holds, then

$$\left(y_2 - y_1 = \int_{x_1}^{x_2} \frac{dy}{dx} dx\right) > \left(g_2 - g_1 = \int_{x_1}^{x_2} \frac{dg}{dx} dx\right)$$

Or if condition (21) holds, then

$$\left(y_2 - y_1 = \int_{x_1}^{x_2} \frac{dy}{dx} dx\right) < \left(g_2 - g_1 = \int_{x_1}^{x_2} \frac{dg}{dx} dx\right)$$

Thus, (22) cannot occur. Condition (22) only occurs when $\frac{dy}{dx} = \frac{dg}{dx}$ for all x.
The lemma is proven.
Checking the conditions satisfying Lemma 2 for the two functions (12) are, respectively:

$$f(x,y) = \sqrt{y^2 + (x - x_0)^2} - \sqrt{y^2 + (x - x_1)^2} = d_1;$$

$$f(x,g) = \sqrt{y^2 + (x - x_0)^2} - \sqrt{y^2 + (x - x_2)^2} = d_2;$$

Then

$$\frac{dx}{dy} = \frac{-d_1}{x_1\sqrt{y^2+x^2} - d_1 x} y, \tag{23}$$

$$\frac{dx}{dg} = \frac{-d_2}{x_2\sqrt{y^2+x^2} - d_2 x} g \tag{24}$$

At point $y = y_a$ and $g = y_a$, taking the difference of the two equations (23) and (24):

$$\frac{dx}{dy} - \frac{dx}{dg} = \frac{-d_1 x_2\sqrt{y^2+x^2} + d_1 d_2 x + d_2 x_1\sqrt{y^2+x^2} - d_1 d_2 x}{\left(x_1\sqrt{y^2+x^2} - d_1 x\right)\left(x_2\sqrt{y^2+x^2} - d_2 x\right)} y_a = \frac{\sqrt{y^2+x^2}\,(d_2 x_1 - d_1 x_2)}{\left(x_1\sqrt{y^2+x^2} - d_1 x\right)\left(x_2\sqrt{y^2+x^2} - d_2 x\right)} y_a \tag{25}$$

Since $d_2 x_1 - d_1 x_2 = const$ and all other components of (25) are greater than "0", then sgn $\left(\frac{dx_1}{dy} - \frac{dx_2}{dy}\right) = const$ (26), meaning that conditions (20) and (21) are always satisfied.
Combining Lemma 1 and Lemma 2 with the conditions satisfying Lemma 2 (26), we see that equation (10) will have one positive real solution, and this solution is unique.
Based on this, an algorithm for determining UAV coordinates using 3 standard landmarks will be constructed.

B. Algorithm for determining uav coordinates using 3 standard landmarks

1. Algorithm

Step 1: Determine the arrival times of the standard pulses at the UAV. To avoid the need for pulses to carry information about their origin, let the pulse from landmark 1 be transmitted τ time before the pulse from landmark 2, and the pulse from landmark 2 be transmitted τ time before the pulse from landmark 3, where $\tau = max\left\{\left(\frac{x_1 x_0}{c}\right), \left(\frac{x_2 x_1}{c}\right)\right\}$; $\tau \geq max\left(\frac{x_1 - x_0}{c}.\frac{x_2 - x_0}{c}\right)$. The arrival times are t_0, t_1, t_2.
Step 2: Determine the distance difference between the UAV and the first landmark with respect to the second and third landmarks: $\Delta l_1 = (t_1 - t_0 - \tau).c$, $\Delta l_2 = (t_2 - t_0 - 2\tau).c$
Step 3: Solve the system of equations (10) to find the solutions x_{a1}, x_{a2} . Substitute x_{a1} and x_{a2} into equation (7) to determine d_a. Choose the pair of solutions x_{ai},d_a that are positive real numbers, d_a>0, and denote x_{ai} as x_a.
Step 4: Determine the altitude ha using a barometric altimeter.
Step 5: Determine the coordinate y_a using the formula:

$$y_a = \sqrt{d_a^2 - (h_a - h_0)^2}$$

The UAV's coordinates will be $M(x_a, y_a, h_a)$.

2. Example Implementation of Algorithm 1

Example 1:
Step 1: Assume the standard points are $D_0(0;0); D_1(20.000;0); D_2(40.000;0)$. The actual coordinates of the UAV are $M_1(5000;10000)m$. According to (26), choose $= \frac{42}{300} = 0{,}14ms$. Determine the UAV coordinates for two cases with different

time measurement accuracies.

a). Time of signal propagation from landmarks 0, 1, 2 to the target are respectively: (Time measured with an accuracy of 0.1 ns)
$t_0(\mu s) = 37{,}2678;\ t_1(\mu s) = 60{,}0925 + 1.400;\ t_2(\mu s) = 121{,}3355 + 2.800$

Step 2: Determine the distance difference $\left(c = \frac{0{,}3km}{\mu s}\right)$

$\Delta l_1 = (t_1 - t_0 - \tau).c = 22{,}8247.0{,}300 = 6{,}84741(km)$
$\Delta l_2 = (t_2 - t_0 - \tau) * c = 84{,}0674 * 0{,}300 = 25{,}22022(km)$

Step 3: Solve equation (10) to determine x_a and from that determine d_a. Obtained
$x_a = 5{,}000020439;\ d_a = 9{,}9999$

Step 4, 5: Assume the barometric altitude is $h_a = h_0$, equal to the altitude of the transmitting pole, then:

$$y_a = \sqrt{d_a^2 - (h_a - h_0)^2} = d_a = 9{,}9999$$

b). Time of signal propagation from landmarks 0, 1, 2 to the target are respectively: (Time measured with an accuracy of 10 ns)
$t_0(\mu s) = 37{,}26;\ t_1(\mu s) = 60{,}09 + 140;\ t_2(\mu s) = 121{,}33 + 280$

Step 2: Determine the distance difference

$$\Delta l_1 = (t_1 - t_0 - \tau).c = 22{,}83.0{,}300 = 6{,}849(km)$$
$$\Delta l_2 = (t_2 - t_0 - \tau) * c = 84{,}07 * 0{,}300 = 25{,}221(km)$$

Step 3: Solve equation (10) to determine x_a and from that determine d_a. Obtained

$x_a = 4{,}9984;\ d_a = 10.0013$

Step 4, 5: Assume the barometric altitude is $h_a = h_0$, equal to the altitude of the transmitting pole, then:

$$y_a = \sqrt{d_a^2 - (h_a - h_0)^2} = d_a = 10.0013$$

Remarks: If time is measured with an accuracy of 0.1 ns, the position error is (0; 0.1) m. If time is measured with an accuracy of 10 ns, the position error is (1.6; 1.3) m.

Example 2*:*

Step 1: Assume the standard points are $D_0(0{,}0); D_1(20.000{,}0); D_2(40.000{,}0)$. The actual coordinates of the target are $M_1(7000{,}15000)$. According to (26), choose $\tau = \frac{42}{300} = 0{,}14ms$. The time of signal propagation from landmarks 0, 1, 2 to the target are respectively:
$t_0(\mu s) = 55{,}1765;\ t_1(\mu s) = 66{,}1648 + 140;\ t_2(\mu s) = 120{,}8305 + 280$

Step 2: Determine the distance difference

$$\Delta l_1 = (t_1 - t_0 - \tau).c = 10{,}9883.300 = 3{,}29649(km)$$

$$\Delta l_2 = (t_2 - t_0 - \tau) * c = 65{,}6540 * 300 = 19{,}6962(km)$$

Step 3: Solve equation (10) to determine x_a and from that determine d_a. Obtained
$x_a = 7{,}000001455;\ d_a = 14{,}9997$

Step 4, 5: Assume the barometric altitude is $h_a = h_0$, equal to the altitude of the transmitting pole, then:

$$y_a = \sqrt{d_a^2 - (h_a - h_0)^2} = d_a = 14{,}9997$$

Conclusions

The algorithms and specific examples presented demonstrate that the method of determining UAV coordinates from 3 standard transmitting landmarks is a simple, easy-to-apply solution that ensures the stability of the mobile object's coordinate determination process for various control purposes. The proposed method can be applied in UAVs or other aerial vehicles when synchronized infrastructure systems are available. However, to refine this method, further research is needed on the anti-jamming capability of transmitted signals, both technically and tactically, and to comprehensively evaluate the method's error.

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