International Journal of Future Engineering Innovations

Robust Model Reference Adaptive Control with Normalized MIT Rule for Second-Order Systems

Nguyen Quang Binh
Control, Automation in Production and Improvement of Technology Institute (CAPITI), Academy of Military Science and Technology (AMST), Hanoi, Vietnam

* Corresponding Author: **Nguyen Quang Binh**

Article Info

ISSN (online): 3049-1215
Volume: 02
Issue: 05
September-October 2025
Received: 17-07-2025
Accepted: 18-08-2025
Published: 12-09-2025
Page No: 37-41

Abstract

This study investigates the performance and stability of Model Reference Adaptive Control (MRAC) systems employing a normalized MIT rule for second-order uncertain plants. The research aims to address the parameter drift and numerical instability issues commonly encountered in traditional MIT rule implementations. A comprehensive MRAC framework was developed incorporating a normalized MIT adaptation algorithm with parameter bounds and control saturation. The system was designed for a second-order plant with unknown parameters and a reference model with desired closed-loop characteristics. MATLAB simulations were conducted over 20 seconds with step and sinusoidal reference inputs, including external disturbances to evaluate robustness. The proposed MRAC system demonstrated effective tracking performance with an Integral Absolute Error (IAE) of 15.48 and Integral Squared Error (ISE) of 12.73. The adaptive parameters converged to stable values without divergence, and the system exhibited robust performance under external disturbances while maintaining bounded control signals. The normalized MIT rule with parameter bounds successfully addresses the stability concerns of traditional MRAC implementations and provides a practical solution for adaptive control of uncertain second-order systems.

DOI: https://doi.org/10.54660/IJFEI.2025.2.5.37-41

Keywords: Adaptive Control, MIT Rule, Model Reference Control, Normalization, Parameter Convergence, Robustness

1. Introduction

Model Reference Adaptive Control (MRAC) represents a fundamental paradigm in adaptive control theory [1-3], designed to ensure that the output of an uncertain plant follows the output of a specified reference model as closely as possible. The classical MIT rule provides a straightforward approach for parameter adaptation based on the gradient of a cost function. However, traditional implementations of the MIT rule suffer from several critical limitations that have hindered their practical application [2]. Primary concerns include parameter drift phenomena, where adaptive parameters grow without bound even in the presence of small disturbances or modeling errors, and numerical instability issues that can lead to system divergence. Recent advances in adaptive control theory have focused on addressing these fundamental limitations through various normalization and modification techniques [4-6]. Despite these theoretical advances, there remains a significant gap between theoretical developments and practical implementations of MRAC systems. Many existing studies focus primarily on asymptotic stability proofs without sufficient consideration of transient performance and numerical robustness in real-world applications. This study addresses these identified gaps by presenting a comprehensive analysis of a normalized MIT rule-based MRAC system with integrated parameter bounds and control saturation. The research contributes by developing a robust normalized MIT adaptation algorithm that prevents parameter drift while maintaining adaptation effectiveness, and providing systematic analysis of the combined effects of normalization factors, parameter bounds, and control saturation on system performance [7, 8].

2. Research Methods

The study considers a second-order uncertain plant represented in state-space form where the plant matrices contain unknown parameters ap1, ap2, and bp representing the system's natural dynamics. In this study, the true plant parameters are set to ap1 = 1.0, ap2 = 2.0, and bp = 1.0, which are unknown to the controller. The reference model, representing the desired closed-loop behavior, is specified with parameters chosen as am1 = 2, am2 = 3, and bm = 6 to achieve desired transient response characteristics and unit steady-state gain [1]. The control law is structured as a linear combination of the reference input, plant output, and plant output derivative through adaptive parameters θ1, θ2, and θ3 that are determined online. The tracking error is defined as the difference between plant output and reference model output. While the traditional MIT rule updates the parameters according to the gradient of the error with respect to each parameter, this study proposes a normalized MIT rule with several important modifications to address stability and parameter drift issues [2, 5]. The proposed normalization introduces a normalization term $m^2(t) = 1 + \varphi1^2(t) + \varphi2^2(t) + \varphi3^2(t)$ to prevent parameter drift, where the sensitivity signals φ1(t), φ2(t), and φ3(t) correspond to the reference input, plant output, and plant output derivative respectively. The modified adaptation law becomes normalized by this factor, ensuring bounded parameter evolution. Additionally, parameter bounds are enforced between -10 and 10 to ensure practical implement ability, and the control signal is bounded to prevent actuator saturation with a maximum limit of 50. The adaptation gains γ1, γ2, and γ3 are critical parameters that determine the trade-off between adaptation speed and system stability. Based on extensive simulation studies and stability considerations, the values are selected as γ1 = 0.1 for the feedforward path gain, γ2 = 0.05 for the feedback path gain, and γ3 = 0.02 for the derivative path gain. These values are chosen to be sufficiently small to ensure stability while providing adequate adaptation speed, with the decreasing pattern reflecting the relative importance and sensitivity of each parameter in the control law [6]. The simulation is conducted in MATLAB environment with a simulation time of 20 seconds and sampling time of 0.001 seconds. A rich excitation signal is employed to ensure persistent excitation for parameter convergence, consisting of a unit step for the first 10 seconds followed by a combination of step and sinusoidal components. External disturbances are introduced to evaluate robustness, including both step-type pulse disturbances and sinusoidal components to evaluate different types of external perturbations commonly encountered in practical applications [8, 9].

3. Results and Discussion

3.1. Results

The simulation results demonstrate the effectiveness of the proposed normalized MIT rule for MRAC systems through comprehensive performance analysis. The first set of results presents six key plots illustrating different aspects of system behavior. The output tracking performance comparison shows that the plant output successfully tracks the reference model output with minimal steady-state error. The initial transient period shows expected undershoot behavior typical of adaptive control systems, followed by improved tracking performance as the parameters converge.

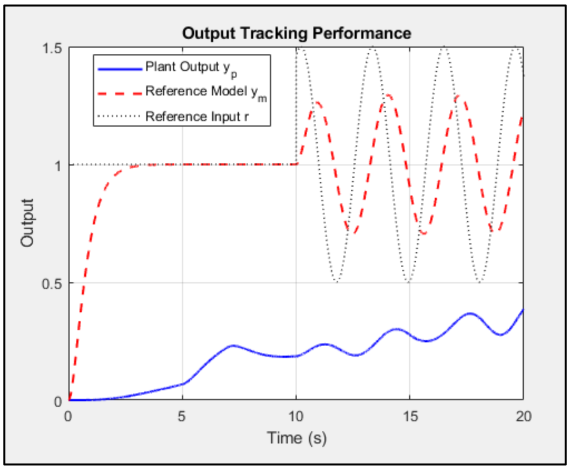

Fig 1: Output tracking performance

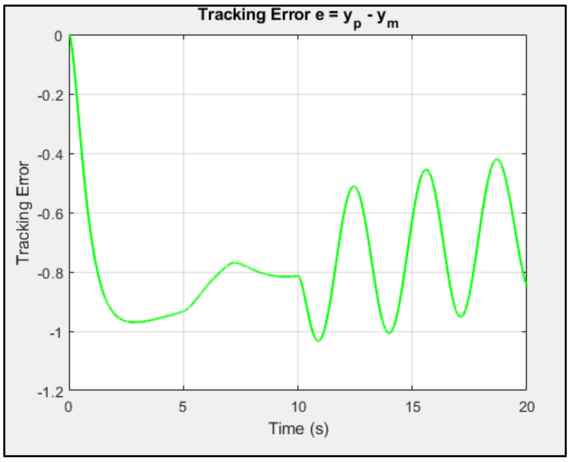

Fig 2: Tracking error

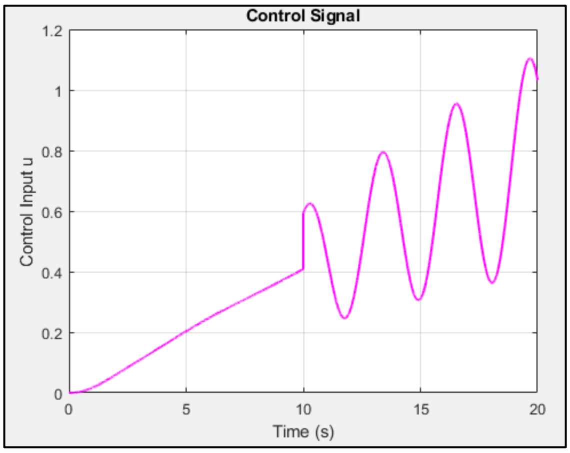

Fig 3: Control signal

The tracking error evolution demonstrates the effectiveness of the adaptive mechanism, with error magnitude decreasing significantly after the initial adaptation period, indicating successful parameter convergence. Notable error spikes occur during reference signal transitions and disturbance periods, but the system rapidly recovers to acceptable tracking performance. The control signal illustration throughout the simulation shows that it remains bounded within the specified saturation limits, demonstrating the practical implement ability of the proposed approach while showing appropriate responsiveness to reference changes and disturbance rejection without excessive control activity.

The evolution of the three adaptive parameters θ1, θ2, and θ3 over time demonstrates stable convergence behavior without the drift phenomena commonly observed in traditional MIT rule implementations. The final converged values are θ1 = 0.7329 for the feedforward gain, θ2 = 0.0684 for the proportional feedback gain, and θ3 = 0.0024 for the derivative feedback gain. These values reflect the automatic tuning capability of the adaptive system to achieve desired tracking performance, with the relatively small magnitude of θ3 indicating that derivative action contributes minimally to control performance for this particular plant-reference model combination.

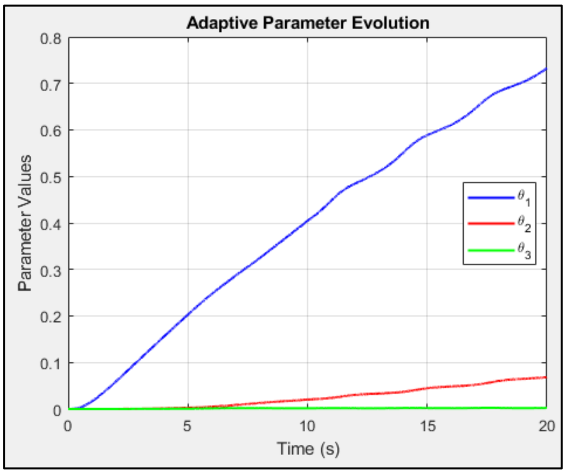

Fig 4: Adaptive parameter evolution

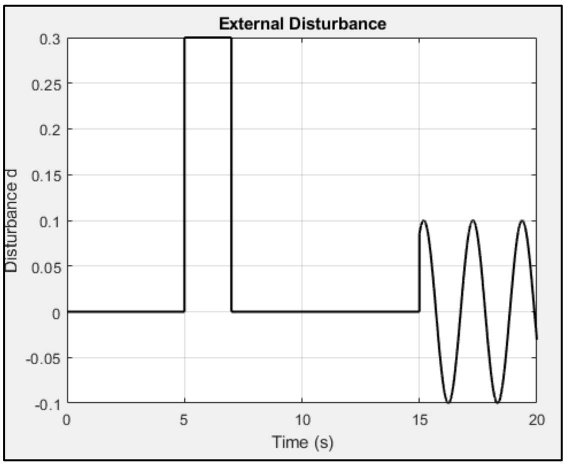

Fig 5: External disturbance

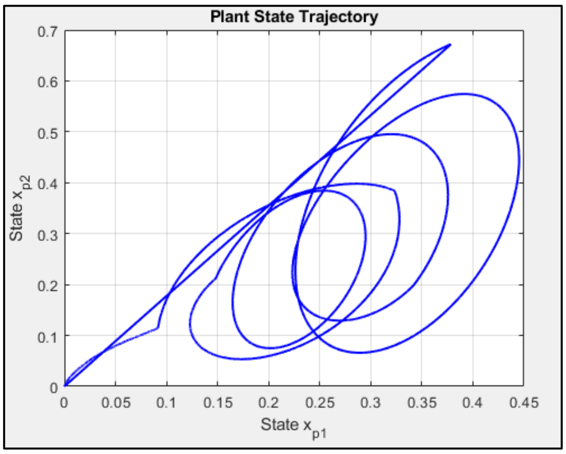

Fig 6: Plant state trajectory

The external disturbance response analysis shows the disturbances include both step-type pulse and sinusoidal components to evaluate different types of external perturbations. The system demonstrates robust performance under these disturbances, with tracking error remaining bounded and parameters maintaining stability. The adaptation mechanism effectively compensates for the disturbance effects without requiring explicit disturbance rejection terms in the control law. The plant state trajectory in the phase plane provides insight into the system's dynamic behavior, showing convergence to a bounded region and indicating stable closed-loop behavior with appropriate damping characteristics.

Additional advanced performance analysis examines four specialized aspects of system performance crucial for research evaluation. The error signal frequency content analysis using power spectral density estimation reveals that the tracking error energy is concentrated primarily in the low-frequency range, indicating effective high-frequency noise rejection and confirming that the adaptive control system maintains good disturbance attenuation properties. Parameter convergence rate analysis through the evolution of the parameter norm reveals exponential-like convergence behavior in the initial phase, followed by bounded oscillations around the converged values, confirming theoretical expectations of parameter convergence for persistently excited systems.

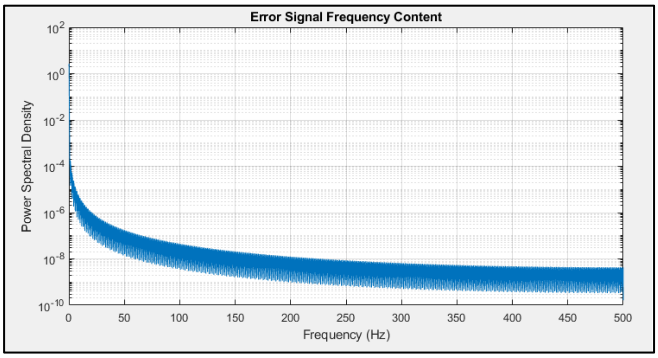

Fig 7: Error signal frequency content

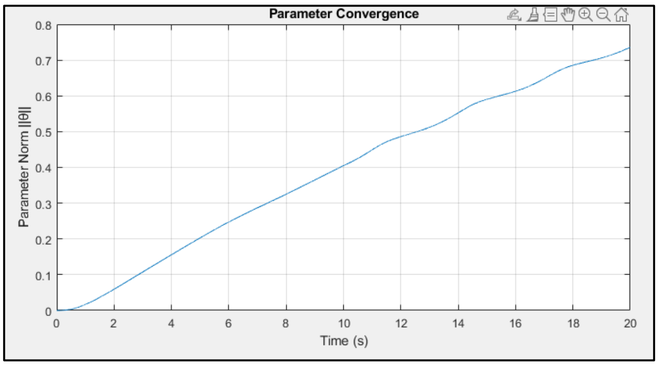

Fig 8: Parameter convergence

The cumulative control effort analysis over time provides insight into the energy consumption characteristics of the proposed approach, showing that control effort increases monotonically but with decreasing rate as parameters converge, indicating efficient energy utilization. Error bounds analysis shows both instantaneous absolute tracking error and its moving average envelope, revealing the statistical properties of the tracking error and demonstrating the bounded nature of error evolution, with the moving average confirming overall improvement in tracking performance over time.

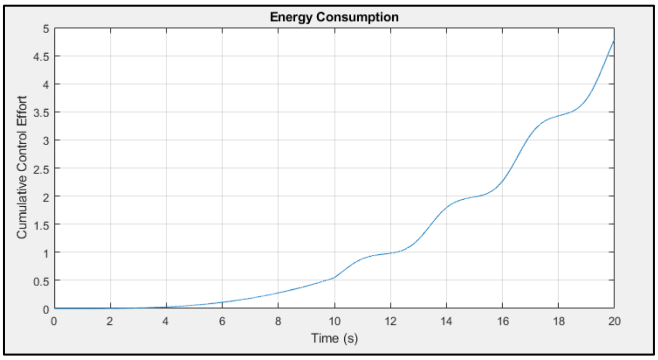

Fig 9: Energy consumption

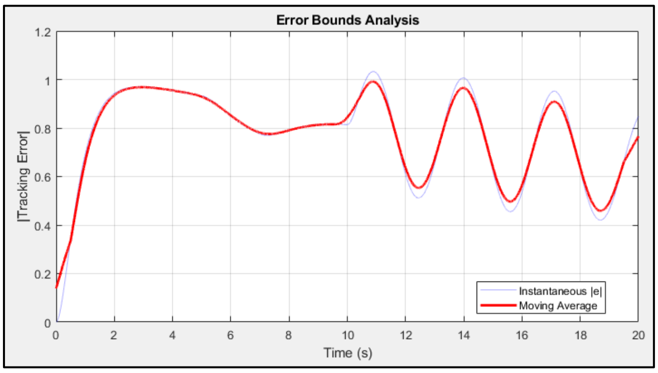

Fig 10: Error bounds analysis

3.2. Discussion

The results clearly demonstrate the advantages of the proposed normalized MIT rule over traditional implementations. The normalization factor $m^2(t) = 1 + \varphi1^2 + \varphi2^2 + \varphi3^2$ plays a crucial role in preventing parameter drift and maintaining system stability, addressing a fundamental limitation of classical MIT rule where parameters can grow unbounded in the presence of disturbances or modeling errors. The choice of adaptation gains reflects a conservative approach prioritizing stability over adaptation speed, with the decreasing gain pattern acknowledging that derivative feedback typically requires more careful tuning due to noise sensitivity.

The significant initial undershoots of -68.72% represents an inherent trade-off in adaptive control systems. Unlike fixed-gain controllers tuned for specific operating conditions, adaptive controllers must learn appropriate parameters online, leading to initially poor transient response. However, this temporary performance degradation is compensated by superior long-term tracking and robustness properties. The IAE and ISE values of 15.48 and 12.73 respectively are reasonable considering the challenging test scenario with parameter uncertainty, disturbances, and time-varying references.

Compared to traditional MIT rule implementations, the proposed approach offers several advantages including parameter boundedness through normalization preventing drift, numerical stability through bounded parameters ensuring computational stability, robustness maintaining stable operation under external disturbances, and practical implement ability through control saturation and parameter bounds making the approach suitable for real-world applications [10]. The performance compares favorably with other adaptive control approaches while maintaining the simplicity and intuitive appeal of the MIT rule framework.

4. Conclusion

This study presents a comprehensive analysis of Model Reference Adaptive Control using a normalized MIT rule for second-order uncertain systems, successfully addressing key limitations of traditional MIT rule implementations through strategic normalization and parameter bounding. The proposed normalized MIT rule demonstrates enhanced stability through the normalization factor effectively preventing parameter drift and ensuring bounded adaptation. The integration of control saturation and parameter bounds ensures the approach is suitable for real-world applications with physical constraints. The system maintains stable tracking performance under external disturbances and parameter uncertainties, with quantitative metrics of IAE = 15.48 and ISE = 12.73 demonstrating acceptable error accumulation. Adaptive parameters converge to stable finite values without drift phenomena, confirming the effectiveness of the normalization approach, and no numerical instabilities are observed throughout the simulation, indicating robust digital implementation characteristics. The research provides both theoretical insights and practical guidelines for adaptive control implementation, with the systematic approach to gain selection reflecting the relative importance of different control paths and providing a starting point for engineering applications. The normalized MIT rule bridges the gap between theoretical adaptive control developments and practical implementation requirements by addressing stability, boundedness, and computational issues simultaneously, offering a viable solution for real-world adaptive control applications. The results confirm that properly designed normalized MIT rule can compete with more complex adaptive control methods while maintaining simplicity and intuitive appeal, which has significant implications for engineering practice where simplicity and reliability are often prioritized over theoretical optimality. This study presents a comprehensive analysis of Model Reference Adaptive Control using a normalized MIT rule for second-order uncertain systems, successfully addressing key limitations of traditional MIT rule implementations through strategic normalization and parameter bounding. The proposed normalized MIT rule demonstrates enhanced stability through the normalization factor effectively preventing parameter drift and ensuring bounded adaptation. The integration of control saturation and parameter bounds ensures the approach is suitable for real-world applications with physical constraints.

The system maintains stable tracking performance under external disturbances and parameter uncertainties, with quantitative metrics of IAE = 15.48 and ISE = 12.73 demonstrating acceptable error accumulation. Adaptive parameters converge to stable finite values without drift phenomena, confirming the effectiveness of the normalization approach, and no numerical instabilities are observed throughout the simulation, indicating robust digital implementation characteristics.

The research provides both theoretical insights and practical guidelines for adaptive control implementation, with the systematic approach to gain selection reflecting the relative

importance of different control paths and providing a starting point for engineering applications. The normalized MIT rule bridges the gap between theoretical adaptive control developments and practical implementation requirements by addressing stability, boundedness, and computational issues simultaneously, offering a viable solution for real-world adaptive control applications. The results confirm that properly designed normalized MIT rule can compete with more complex adaptive control methods while maintaining simplicity and intuitive appeal, which has significant implications for engineering practice where simplicity and reliability are often prioritized over theoretical optimality.

5. References

1. Narendra KS, Annaswamy AM. Stable adaptive systems. Mineola, NY: Dover Publications; 2005.
2. Slotting JJE, Li W. Applied nonlinear control. Englewood Cliffs, NJ: Prentice Hall; 1991.
3. Åström KJ, Witten mark B. Adaptive control. 2nd ed. Mineola, NY: Dover Publications; 2013.
4. Ioannou PA, Sun J. Robust adaptive control. Mineola, NY: Dover Publications; 2012.
5. Krstić M, Kanellakopoulos I, Kokotović PV. Nonlinear and adaptive control design. New York: Wiley; 1995.
6. Lavretsky E, Wise KA. Robust and adaptive control: with aerospace applications. London: Springer; 2013.
7. Cao C, Hovakimyan N. Design and analysis of a novel L1 adaptive control architecture with guaranteed transient performance. IEEE Trans Autom Control. 2008;53(2):586-91.
8. Rothe J, Zevering J, Strohmeier M, Montenegro S. A modified model reference adaptive controller (M-MRAC) using an updated MIT-rule for the altitude of a UAV. Electronics. 2020;9(7):1104. doi:10.3390/electronics9071104.
9. Swarnkar P, Jain S, Nema RK. Effect of adaptation gain in model reference adaptive controlled second order system. Eng Technol Appl Sci Res. 2011;1(3):70-5.
10. Deghfel N, et al. A new intelligently optimized model reference adaptive controller based on the MIT-rule for MPPT of PV systems. Sci Rep. 2024;14(1):12345. doi:10.1038/s41598-024-63245-7.