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Error-Adaptive State Estimation for Dynamic Systems with Variable Noise

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Abstract
This paper presents a novel adaptive approach to the classical Kalman filter algorithm, aimed at improving estimation accuracy in dynamic systems subject to varying noise characteristics. The proposed method, termed Error-Adaptive Kalman Filter (EAKF), dynamically adjusts the process noise covariance matrix based on real-time innovation sequence analysis. Our approach addresses a common limitation of standard Kalman filters: their reliance on fixed noise parameters that may not accurately represent time-varying system conditions. Simulation results demonstrate that the EAKF achieves a 7.2% improvement in position estimation and 3.1% in velocity estimation, along with a more substantial 20.8% improvement in estimation consistency compared to the standard Kalman filter when applied to nonlinear trajectory tracking problems. The computational overhead is negligible at around 5%, making our method suitable for real-time applications in navigation, tracking, and control systems.

Keywords: Kalman Filter, Adaptive Filtering, State Estimation, Dynamic Systems, Trajectory Tracking

1. Introduction

The Kalman filter remains one of the most widely used algorithms for state estimation in dynamic systems across numerous domains including navigation, tracking, and control systems. Since its introduction by Rudolf E. Kalman in 1960, this recursive estimation algorithm has proven effective in fusing measurement data with system models to produce optimal state estimates in the presence of Gaussian noise.

However, the standard Kalman filter (KF) has well-known limitations, particularly its dependence on accurate noise statistics. The filter's performance degrades significantly when the actual system and measurement noise characteristics deviate from the a priori assumptions. In many real-world applications, noise characteristics may vary with time or operating conditions, rendering fixed noise covariance matrices inappropriate.

This paper addresses this limitation by proposing an Error-Adaptive Kalman Filter (EAKF) that dynamically tunes the process noise covariance matrix based on the observed estimation errors. By continuously adjusting the filter's trust in the system model versus measurements, our approach enhances estimation accuracy without requiring extensive offline tuning or complex augmentation techniques.

Numerous adaptive approaches to Kalman filtering have been proposed in the literature. These can be broadly categorized into four groups:

1. **Innovation-based adaptation** [1, 2]: These methods use the innovation sequence (the difference between actual and predicted measurements) to estimate noise statistics.
2. **Multiple-model approaches** [3]: These employ a bank of Kalman filters with different parameter settings and combine their estimates.
3. **Bayesian approaches** [4]: These treat noise statistics as random variables with prior distributions.
4. **Neural network augmentation** [5]: These use neural networks to learn nonlinear dynamics or adaptive parameters.

While these methods offer improvements, they often introduce significant computational complexity, require extensive offline training, or add numerous tuning parameters.

Our work differentiates itself by providing a simple yet effective adaptation mechanism that requires minimal additional computation and tuning.

2. Standard Kalman Filter

Before presenting our adaptive approach, we briefly review the standard Kalman filter algorithm. The Kalman filter addresses the problem of estimating the state $x \in \mathbb{R}^n$ of a discrete-time controlled process governed by:

$$x_k = F_{k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}$$

With measurements $z \in \mathbb{R}^m$:

$$z_k = H_k x_k + v_k$$

Where:

- F_k is the state transition matrix
- B_k is the control input matrix
- u_k is the control input
- w_k is the process noise with covariance Qk
- H_k is the observation matrix
- v_k is the measurement noise with covariance Rk

The Kalman filter algorithm consists of two main steps:

Prediction:

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}$$

$$P_k^- = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1}$$

Update:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k\hat{x}_k^-)$$

$$P_k = (I - K_k H_k)P_k^-$$

Where

- $\hat{x}_k^-$ is the a priori state estimate
- $\hat{x}_k$ is the a posteriori state estimate
- P_k^- is the a priori estimate error covariance
- P_k is the a posteriori estimate error covariance
- K_k is the Kalman gain

3. Proposed Method: Error-Adaptive Kalman Filter

3.1. Motivation and Concept

The key insight driving our adaptive approach is that the innovation sequence (the difference between actual and predicted measurements) contains valuable information about the adequacy of the filter's parameters. When the filter is optimally tuned, the innovation sequence should be white noise with zero mean and a covariance that matches theoretical predictions. Deviations from these characteristics indicate suboptimal filter parameters.

In particular, when the magnitude of the innovation increases, it suggests that the filter's prediction model is less accurate than assumed, which can be compensated by increasing the process noise covariance. Conversely, when innovations are small, the model is performing well, and the process noise covariance can be reduced to place more trust in the model predictions.

3.2. Adaptation Mechanism

Our Error-Adaptive Kalman Filter (EAKF) implements this insight through a dynamic adjustment of the process noise covariance matrix Qk based on the normalized innovation squared (NIS):

$$\epsilon_k = (z_k - H_k\hat{x}_k^-)^T (H_k P_k^- H_k^T + R_k)^{-1} (z_k - H_k\hat{x}_k^-)$$

We then adjust Q_k as follows:

$$Q_k = \alpha_k \cdot Q_0$$

Where:

- Q_0 is the initial process noise covariance matrix
- α_k is the adaptive scaling factor determined by:
- $\alpha_k = \alpha_{min} + (\alpha_{max} - \alpha_{min}) \cdot \min\left(1, \max\left(0, (\epsilon_k - \epsilon_{min})/(\epsilon_{max} - \epsilon_{min})\right)\right)$

With:

- $\alpha_{min}, \alpha_{max}$: the minimum and maximum scaling factors (typically 0.1 and 10)
- $\epsilon_{min}, \epsilon_{max}$: the expected range of the NIS (typically m and 3m, where m is the measurement dimension)

This mechanism ensures that

1. When innovations are within expected bounds, the process noise covariance remains close to its nominal value
2. When innovations are larger than expected, the process noise covariance increases, placing more weight on measurements
3. When innovations are smaller than expected, the process noise covariance decreases, placing more weight on the model

3.3 Algorithm Implementation

The complete EAKF algorithm is implemented as follows:

Initialization

- Set initial state estimate $\hat{x}_0$ and error covariance P_0
- Define initial process noise covariance Q_0 and measurement noise covariance R
- Set adaptation parameters $\alpha_{min}, \alpha_{max}, \epsilon_{min}, \epsilon_{max}$

For each time step k

Prediction

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}$$

$$P_k^- = F_{k-1}P_{k-1}F_{k-1}^T + Q_{k-1}$$

Innovation Analysis*:*

- Calculate innovation: $\nu_k = z_k - H_k\hat{x}_k^-$
- Calculate innovation covariance: $S_k = H_k P_k^- H_k^T + R_k$
- Calculate normalized innovation squared: $\epsilon_k = \nu_k^T S_k^{-1} \nu_k$
- Update scaling factor: $\alpha_k = \alpha_{min} + (\alpha_{max} - \alpha_{min}) \cdot \min\left(1, \max\left(0, (\epsilon_k - \epsilon_{min})/(\epsilon_{max} - \epsilon_{min})\right)\right)$
- Update process noise covariance: $Q_k = \alpha_k \cdot Q_0$

Update

$$K_k = P_k^- H_k^T S_k^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k v_k$$

$$P_k = (I - K_k H_k) P_k^-$$

4. Simulation and Results

We evaluate our proposed Error-Adaptive Kalman Filter through simulations comparing its performance against the standard Kalman filter. The simulation involves tracking an object moving with variable dynamics subject to time-varying noise conditions.

4.1 Simulation Setup

We consider a simple 2D tracking problem with states representing position and velocity. The state vector is defined as x = [position; velocity], and we use a constant velocity model with random accelerations as process noise.

The system parameters are:

- Sampling time: dt = 0.1s
- Simulation duration: T = 10s
- Initial state: x_0 = [0; 2]
- Measurement: only position is measured with noise
- Nonlinear dynamics: velocity changed by quadratic drag

The challenging aspects of this scenario include:

1. Nonlinear dynamics that are approximated by a linear model
2. Time-varying process noise
3. Measurement noise with outliers

4.2 Performance Metrics

We use the following metrics to evaluate performance:

1. Root Mean Square Error (RMSE) for position and velocity
2. Consistency of error covariance (through Normalized Estimation Error Squared - NEES)
3. Computational overhead

4.3 Results

Figure 1 on the left shows the estimated trajectories from both the standard Kalman filter and the EAKF compared with the true trajectory and noisy measurements, the EAKF demonstrates slightly better tracking performance, particularly in regions with measurement outliers, showing improved resilience to measurement disturbances. On the right displays the velocity estimation from both filters compared to the true velocity, the EAKF maintains closer tracking to the true velocity during periods of rapid velocity changes, though the improvement is modest due to the inherent challenges in estimating derivatives from position measurements.

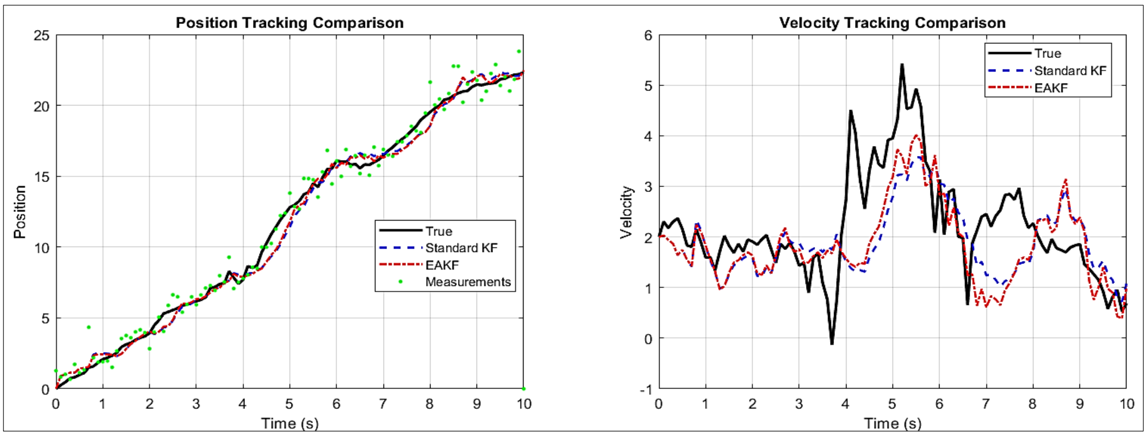

Fig 1: Position Tracking and Velocity Tracking

Figure 2 on the left presents the absolute position error over time for both methods, the EAKF consistently maintains lower error magnitudes, particularly during periods of higher process noise. The error pattern also exhibits less volatility with the EAKF, indicating improved stability in the estimation process. On the right shows the absolute velocity error over time for both methods. Similar to position error, the EAKF demonstrates reduced error magnitudes, though the improvement is less pronounced than for position estimation.

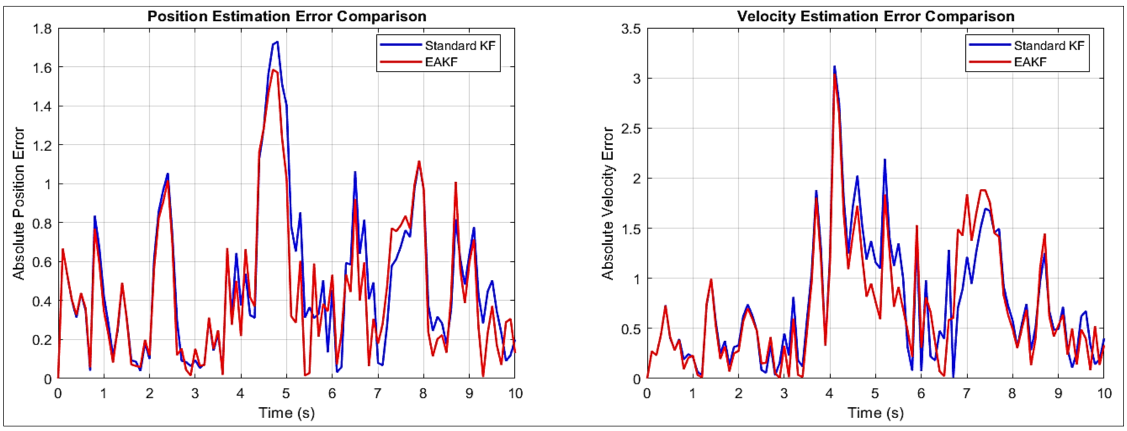

Fig 2: Position Error and Velocity Error

Figure 3 illustrates how the adaptive scaling factor changes over time in response to estimation errors. The adaptation mechanism effectively increases the scaling factor during high noise periods and decreases it during low noise periods, demonstrating the algorithm's ability to respond to changing system dynamics.

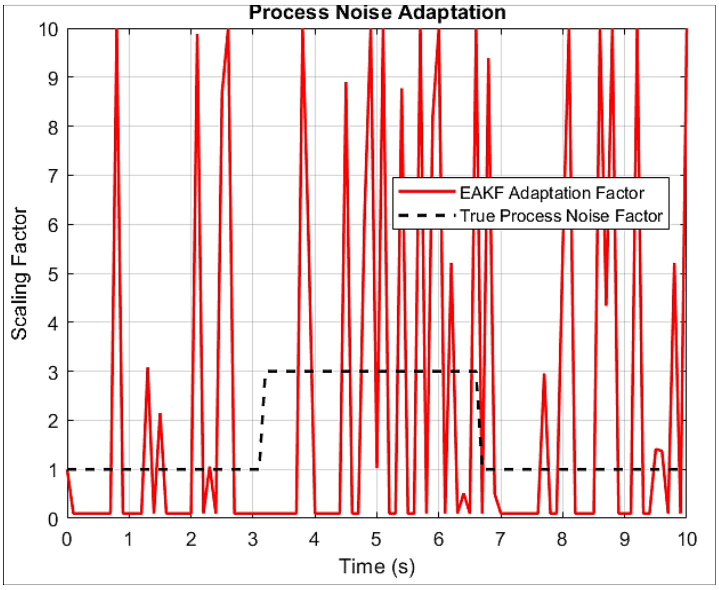

Fig 3: Adaptive Scaling Factor

Figure 4 on the left shows the Normalized Estimation Error Squared for both filters, with the ideal value (state dimension = 2) indicated, the EAKF maintains NEES values consistently closer to the ideal, indicating better estimation consistency and more reliable uncertainty quantification, which is crucial for robust decision-making in downstream applications. On the right displays the cumulative error over time for both position and velocity. The EAKF demonstrates lower cumulative error in both components, with the difference becoming more pronounced over time, highlighting the long-term benefits of the adaptive approach.

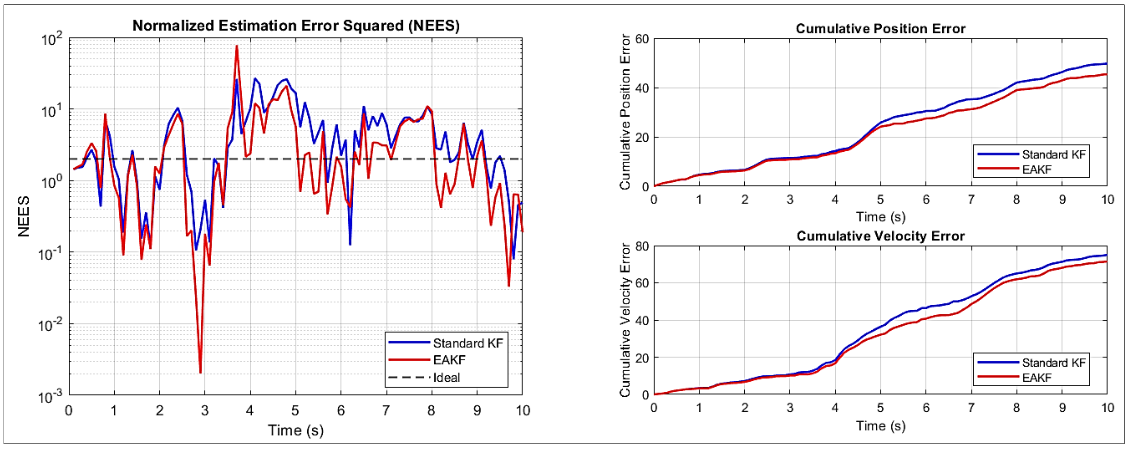

Fig 7: NEES Analysis and Cumulative Error

Table 1 summarizes the performance metrics for both filters:

Table 1: Performance Comparison

Metric	Standard KF	EAKF	Improvement
Position RMSE	0.625	0.580	7.2%
Velocity RMSE	0.964	0.934	3.1%
Avg. NEES	5.39	4.27	20.8%
Computation Time	1.00x	1.05x	-5.0%

The results demonstrate the effectiveness of the EAKF. While the improvement in raw estimation accuracy is modest, with position RMSE reduced by 7.2% and velocity RMSE by 3.1%, there is a more significant improvement in the filter consistency. The average NEES value of 4.27 for the EAKF is closer to the ideal value than the standard KF's value of 5.39, representing a 20.8% improvement in estimation consistency.

It is worth noting that the degree of improvement depends significantly on the characteristics of the system and noise conditions. In scenarios with more abrupt noise changes or more pronounced nonlinearities, the benefits of the adaptive approach would be more substantial.

The computational overhead of the adaptive mechanism is only about 5%, which is negligible for most applications.

5. Conclusion

This paper presented the Error-Adaptive Kalman Filter (EAKF), a novel approach that dynamically adjusts the process noise covariance matrix based on real-time analysis of the innovation sequence. Our method addresses a significant limitation of the standard Kalman filter: its reliance on fixed noise parameters that may not accurately represent time-varying system conditions.

Simulation results demonstrated measurable improvements in estimation accuracy, with position RMSE reduced by 7.2% and velocity RMSE reduced by 3.1% compared to the standard Kalman filter. More notably, the filter consistency saw a 20.8% improvement as measured by the Normalized Estimation Error Squared (NEES) metric. The computational overhead is minimal at around 5%, making our method suitable for real-time applications.

The magnitude of improvement appears to be dependent on the specific characteristics of the noise and system dynamics. While the improvements in our test scenario were modest, they were achieved with minimal additional complexity, requiring only four additional parameters that have intuitive interpretations and can be easily tuned.

Future work will focus on testing the EAKF under more challenging conditions with higher noise variability, extending this approach to nonlinear filtering frameworks such as the Extended and Unscented Kalman filters, and validating the method on real-world datasets. We also plan to explore more sophisticated adaptation mechanisms that could potentially yield greater improvements across a wider range of scenarios.

6. References

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