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# Overview of Modeling and Solution Methods for Vehicle Vibration Equations in Vehicle Dynamics

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## Abstract

Vehicle vibration is a core topic in automotive dynamics because it affects ride comfort, road holding, structural durability, and driver health. A reliable vibration study normally requires two steps: selecting a suitable dynamic model and choosing an appropriate method to solve the governing vibration equations. This paper reviews common modeling approaches and solution methods used in vehicle vibration analysis, including quarter-car, half-car, full-vehicle, and extended cab-seat-driver models. Analytical solutions, transfer-function analysis, state-space formulation, numerical integration, MATLAB/Simulink simulation, frequency-domain analysis, and simulation-based optimization are discussed. The review shows that simple models remain useful for understanding basic vibration mechanisms, whereas full-vehicle and multi-degree-of-freedom models are more suitable for evaluating ride comfort, dynamic tire load, cab vibration, and seat vibration under practical operating conditions. Recent studies also show that optimization algorithms, including genetic algorithms and swarm-based methods, are increasingly used to improve suspension parameters and control performance, especially for electric vehicles.

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**Keywords:** vehicle vibration, automotive dynamics, vibration equations, suspension system, ride comfort, numerical simulation, genetic algorithm

## 1. Introduction

Vehicle vibration is caused by several excitation sources, including road unevenness, tire-road interaction, powertrain vibration, braking and traction forces, wheel imbalance, and aerodynamic disturbances. Among these sources, road roughness is usually the dominant input in vertical vibration analysis. When a vehicle moves over an uneven road surface, the excitation is transmitted through the tires, suspension system, chassis, cab, seat, and finally to the occupants. Excessive vibration may reduce ride comfort, increase dynamic tire load, deteriorate road holding, and shorten the fatigue life of mechanical components [1, 4].

In vehicle dynamics, vibration equations are commonly established using Newton's law, Lagrange's equation, or energy-based approaches. Depending on the research objective, different model levels can be selected, from quarter-car models to half-car, full-vehicle, and multi-degree-of-freedom models. Textbooks on vehicle dynamics have shown that simplified models are valuable for understanding the physical mechanism of suspension vibration, whereas more detailed models are needed when pitch, roll, tire dynamics, and road-vehicle interaction are considered [1, 7].

Another important issue is the evaluation criterion. For ride comfort and human exposure to vibration, ISO 2631-1 is widely used to calculate frequency-weighted acceleration and assess the effect of whole-body vibration on occupants [8]. For road excitation modeling, ISO 8608 provides a standard basis for describing road roughness using power spectral density [9]. Therefore, modern vibration studies often combine vehicle dynamic models, road spectral models, numerical simulation, and comfort assessment criteria.

In recent years, vehicle vibration research has moved from pure response calculation to simulation-based design and optimization. Active and semi-active suspension control methods have been reviewed and developed in many studies [10, 12, 13]. In addition, optimization algorithms have been used to tune suspension parameters and controller gains, especially for electric vehicles and heavy vehicles, where ride comfort and road holding must be balanced [19, 21].

This paper provides an overview of modeling and solution methods for vehicle vibration equations in automotive dynamics, with emphasis on practical model selection and engineering application.

## 2. Modeling Approaches for Vehicle Vibration Analysis

### 2.1. Quarter-car model

The quarter-car model is the simplest and most widely used model in vehicle vibration studies. It normally consists of a sprung mass, an unsprung mass, a suspension spring, a suspension damper, and a tire stiffness element. Although the model represents only one wheel station, it is useful for studying the influence of suspension stiffness, damping coefficient, tire stiffness, and road excitation on vertical vibration response [1, 2, 6, 21].

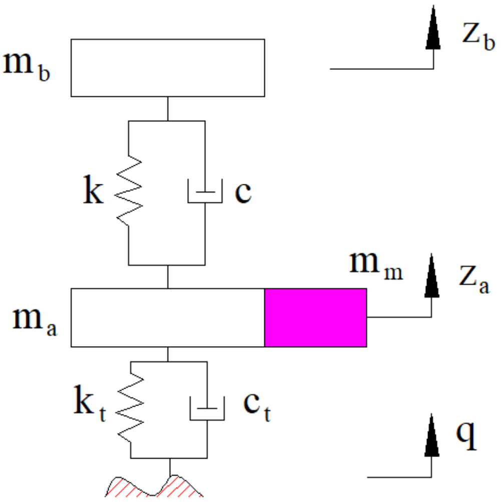

**Fig 1:** Quarter-car dynamic model of the electric vehicle [21]

### 2.2. Half-car model

The half-car model is an extension of the quarter-car model and is commonly used to describe vertical and pitch motions of the vehicle body. This model includes the front and rear unsprung masses, front and rear suspension systems, and the pitch motion of the sprung mass. It can consider the wheelbase effect and the time delay between front and rear road inputs [1, 2, 6].

Compared with the quarter-car model, the half-car model is more suitable for studying longitudinal ride behavior, especially when the pitch response of the vehicle body is significant. It is often used for passenger cars, buses, trucks, and electric vehicles. However, because the half-car model neglects roll motion and left-right asymmetry, it may not be sufficient for vehicles operating on uneven off-road surfaces or under asymmetric road excitation.

### 2.3. Full-vehicle model

A full-vehicle vibration model generally includes bounce, pitch, and roll motions of the sprung mass, together with the vertical motions of four unsprung masses. A common full-car model has seven degrees of freedom, but more detailed models may include the powertrain, cab, seat, driver body, or flexible body components [1, 2, 6].

The full-vehicle model is important because practical vehicle vibration is rarely limited to pure vertical motion. When a vehicle travels on a random road, the four wheels may receive different excitations. As a result, the vehicle body can simultaneously experience vertical, pitch, and roll vibrations. Therefore, full-vehicle models are more suitable for evaluating ride comfort, road friendliness, and suspension performance under realistic operating conditions [7, 9].

For heavy trucks, buses, off-road vehicles, and construction machinery, three-dimensional dynamic models are especially useful. Previous studies have shown that dynamic parameters of heavy trucks, including suspension, tire, cab, and seat parameters, can strongly influence ride comfort and whole-body vibration response [14, 18].

### 2.4. Extended cab-seat-driver models

For commercial vehicles, off-road vehicles, and construction machinery, the driver may be exposed to low-frequency vibration for long periods. In these cases, evaluating only the body acceleration is not sufficient. The vibration transmitted to the cab floor, seat base, seat cushion, and driver body should also be considered [8, 14, 18].

Extended cab-seat-driver models provide a more practical basis for assessing vibration isolation performance. The cab suspension and seat suspension act as additional vibration isolation stages. Their stiffness and damping parameters directly affect the weighted RMS acceleration at the seat and the driver’s exposure to whole-body vibration [8, 15, 18]. These models are more complicated than quarter-car and half-car models, but they are more suitable for practical ride comfort design of heavy vehicles and off-road machinery.

## 3. Solution Methods for Vehicle Vibration Equations

### 3.1. Analytical solution

Analytical methods are mainly suitable for simple linear vibration systems. For low-order models with idealized inputs, closed-form solutions can be obtained. These solutions are useful because they clearly show the relationship between system parameters and vibration response. For example, the natural frequency of the sprung mass depends mainly on the suspension stiffness and sprung mass, while the damping ratio depends on the suspension damping coefficient [1, 2, 6].

The main advantage of analytical methods is physical clarity. They help researchers understand the vibration mechanism before moving to complex simulations. However, most practical vehicle vibration problems involve random road inputs, nonlinear damping, tire nonlinearity, or control systems. In such cases, analytical solutions are difficult to obtain and numerical methods become necessary.

### 3.2. Transfer-function and frequency-domain methods

For linear systems, transfer-function analysis is an effective method for studying vehicle vibration in the frequency domain. By applying the Laplace transform to the governing equations, the relationship between road input and vehicle response can be expressed through transfer functions. Typical output responses include body acceleration, suspension deflection, tire deflection, and dynamic tire load [1, 6, 10].

Frequency-domain analysis is particularly useful when road excitation is described by power spectral density. ISO 8608 provides a standard method for classifying road roughness, which can be combined with vehicle transfer functions to calculate response spectra [7, 9]. This approach is efficient for studying the effects of vehicle speed, road class, and suspension parameters on ride comfort. However, it is mainly suitable for linear or linearized systems.

### 3.3. State-space method

The state-space method is widely used in modern vehicle dynamics and control. A second-order vibration system can be rewritten as a first-order matrix equation:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $x$ is the state vector, $u$ is the input vector, $y$ is the output vector, and $A$, $B$, $C$, and $D$ Are system matrices. This formulation is convenient for simulation, stability analysis, observer design, and controller development [10, 12, 13].

State-space models are widely applied in active and semi-active suspension systems. They are suitable for skyhook control, LQR control, fuzzy control, PID control, and hybrid control strategies [10, 12, 13, 18, 19]. For electric vehicles, state-space modeling is also useful because suspension dynamics may be coupled with additional effects from battery mass distribution and motor-related excitation.

### 3.4. Numerical integration and MATLAB/Simulink simulation

Numerical integration is the most practical method for solving vehicle vibration equations with many degrees of freedom, nonlinear elements, or complex excitations. Common methods include Euler, Runge-Kutta, Newmark-beta, and built-in MATLAB solvers such as ode45 and ode15s. MATLAB/Simulink is widely used because it allows dynamic models, controllers, road inputs, and optimization algorithms to be connected in a flexible simulation environment [6, 19, 21].

Time-domain simulation can handle nonlinear suspension characteristics, measured road profiles, random road inputs, and different control strategies. It also allows direct calculation of acceleration, displacement, suspension working space, and tire dynamic load. When combined with ISO 2631-1, the simulated acceleration signal can be filtered and weighted to evaluate ride comfort and human exposure to vibration [8, 14, 18].

### 3.5. Simulation-based optimization

In recent studies, solving vehicle vibration equations is often combined with optimization. The aim is not only to calculate the vibration response but also to search for optimal suspension parameters or controller gains. Common design variables include suspension stiffness, damping coefficient, seat suspension stiffness, cab isolation parameters, hydraulic-pneumatic suspension parameters, and control gains [10, 12, 13, 19, 21].

Genetic algorithms, particle swarm optimization, multi-objective particle swarm optimization, grey wolf optimizer, and other evolutionary algorithms are suitable for suspension optimization because the objective functions are often nonlinear and conflicting. For example, improving ride comfort usually requires reducing body acceleration, while road holding requires controlling dynamic tire load. Therefore, multi-objective optimization is often needed. The work by Dat and Quynh [21] is relevant in this direction because it applies a genetic algorithm to optimize suspension systems for electric vehicles. This reference strengthens the link between vehicle vibration modeling, numerical simulation, and EV suspension optimization.

## 4. Classification of References and Method Selection

Table 1 classifies the reviewed references into five groups: textbooks/standards, modeling, control, optimization, and electric vehicle suspension. This grouping shows how the cited studies support different parts of the review, from basic vehicle dynamics and road excitation modeling to suspension control and optimization.

The selection of a vehicle vibration model and solution method should depend on the research objective. If the purpose is to explain the basic influence of suspension stiffness and damping, a quarter-car model with analytical or transfer-function methods is sufficient [1, 2, 6]. If the objective is to study vertical and pitch responses, a half-car model is more appropriate. If the objective is to evaluate complete ride performance under random road excitation, a full-vehicle model combined with time-domain or frequency-domain simulation should be used [7, 9].

For heavy vehicles and off-road machinery, extended cab-seat-driver models are necessary because the vibration transmitted to the driver is a key performance index [8, 14, 18]. For active and semi-active suspension systems, the state-space method is preferred because it provides a direct framework for control design [10, 12, 13, 19]. For electric vehicle suspension optimization, numerical simulation combined with genetic algorithms or other intelligent optimization methods is a practical and promising approach [19, 21].

**Table 1:** Classification of references used in the review

| Group | References | Main contribution to the review |
|---|---|---|
| Textbooks and standards | [1, 9] | Provide the theoretical foundation of vehicle dynamics, tire dynamics, road-vehicle interaction, road roughness modeling, and whole-body vibration assessment. |
| Vehicle vibration modeling | [1, 7, 14, 18] | Support the discussion of quarter-car, half-car, full-vehicle, cab-seat-driver, and heavy-vehicle vibration models. |
| Control-oriented methods | [10, 12, 13, 18, 19] | Support the discussion of state-space modeling, active suspension, semi-active suspension, fuzzy control, PID control, and controller-based ride comfort improvement. |
| Optimization methods | [10, 19, 21] | Support the use of optimization algorithms for suspension parameter tuning and controller improvement. |
| Electric vehicle suspension | [19, 21] | Provide recent examples related to EV suspension control and GA-based optimization of EV suspension systems. |

For a practical vibration study, the dynamic model should first be selected according to the target response, such as body acceleration, suspension working space, dynamic tire load, or seat-weighted acceleration. The governing equations can then be formulated and solved using analytical, numerical, or simulation-based methods. When the model is used for suspension design, optimization and experimental validation are usually required to improve the reliability of the obtained results.

## 5. Conclusions

This review has summarized the main modeling and solution approaches used for vehicle vibration equations in vehicle dynamics. The quarter-car model remains useful for basic suspension analysis because it clearly shows the effects of suspension stiffness, damping, tire stiffness, and road input on vertical vibration response. For problems involving pitch motion, the half-car model is more suitable, while full-vehicle models are required when bounce, pitch, roll, and four-wheel excitations must be considered together. In heavy vehicles and off-road vehicles, cab-seat-driver models are necessary because the vibration transmitted to the driver is one of the main indicators of ride comfort and whole-body vibration exposure [8, 14, 18].

The reviewed solution methods also have different application ranges. Analytical and transfer-function methods are appropriate for simple linear models and are useful for understanding the physical characteristics of the vibration system [1, 2, 6]. State-space methods are more suitable for active and semi-active suspension control because they can be directly connected with controller design [10, 12, 13]. For nonlinear systems, random road inputs, and multi-degree-of-freedom models, numerical integration and MATLAB/Simulink simulation provide a more flexible approach [6, 19, 21].

For suspension design, the choice of model and solution method should be linked to the target performance indices. Ride comfort, suspension working space, and dynamic tire load often lead to conflicting requirements; therefore, optimization methods are useful for selecting suspension parameters or controller gains. The GA-based study on electric vehicle suspension optimization in [21] is a relevant example showing how vibration modeling can be combined with numerical simulation and parameter optimization. Future work should place more emphasis on experimental validation, parameter identification, and EV-specific factors such as battery mass distribution, motor-related excitation, and their influence on ride comfort.

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